45841

AN ROINN OIDEACHAIS

M.47

LEAVING CERTIFICATE EXAMINATION, 1990

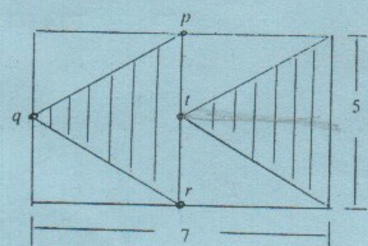
MATHEMATICS – ORDINARY LEVEL – PAPER I (300 marks)

THURSDAY 7 JUNE – MORNING, 9.30 – 12.00

Attempt **Question 1** (100 marks) and **four** other questions (50 marks each)

Marks may be lost if all your work is not clearly shown or if you do not indicate where a calculator has been used

1\. (i) Calculate the area of the shaded portion of the rectangle if p, q, r, and t, are the midpoints of sides.



$17\frac{1}{2}$

$\frac{4}{8}$ 5x7=35 $\frac{1}{8}$

(ii) Express t in terms of a, b, and c when

$$a + \frac{tc}{b} = c.$$

$a + \frac{tc}{b} - c$

$ab + tc = cb$

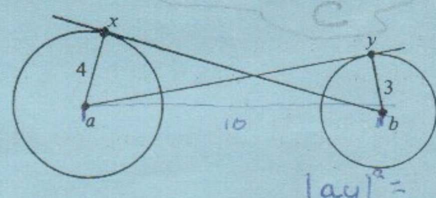
$tc = cb - ab$

$t = \frac{cb - ab}{c}$

(iii) The length of the radius of each circle is shown. ay, bx are tangents.

$|ab| = 10$.

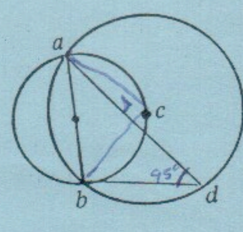
Verify $|ay| > |bx|$.



$|ay|^2 = 10^2 - 3^2$

$|ax|^2 = 10^2 - 4^2$

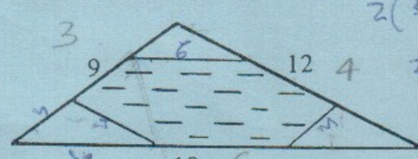
(iv) Two circles intersect in a and b.
$[ab]$ is a diameter.
c is the centre of the other circle.
Calculate $|\angle bda|$.



$\frac{2}{3}(9+12+18)$

$2(3+4+6)$

(v) The lengths of the sides of a triangle are 9, 12, and 18.
Each side is divided into three segments of equal length.
Calculate the length of the perimeter of the shaded portion.



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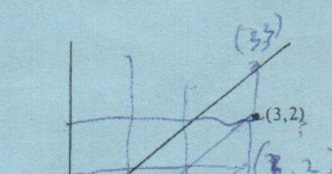
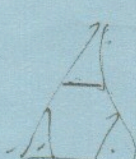
(vi) Write down the coordinates of the image of (3, 2) under the projection of the plane on the X-axis parallel to the line $y = x$.





81

OVER→

$\sqrt{6^2+3^2} = |pq|$ (0, 6)

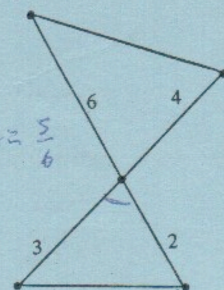
$\sqrt{45} = 3\sqrt{5}$ (3, 0)

(vii) p and q are the points at which the line $2x + y = 6$ intersects the axes. Calculate $|pq|$.

$x^2 + y^2 = 5^2$

(viii) The points $(-5, 0)$, $(0, 5)$, $(5, 0)$ are on a circle. Write out the equation of the circle.

(ix) The area of the smaller of the two triangles shown is $\frac{5}{2}$. Calculate the area of the larger.



$x = \frac{5}{6}$

$\frac{1}{2}\, 3\, 2\, x = \frac{5}{2}$ $\quad 6 \times 4 \times \frac{5}{6} = 20$

$3x = \frac{5}{2}$

(x) If $16\vec{i} + 19\vec{j} = 3(a\vec{i} + b\vec{j}) + (\vec{i} + 2\vec{j})$ find the value of a and the value of b.

$= (3a+1)\vec{i} + (3b+2)\vec{j}$ $\quad a = 5$, $b =$

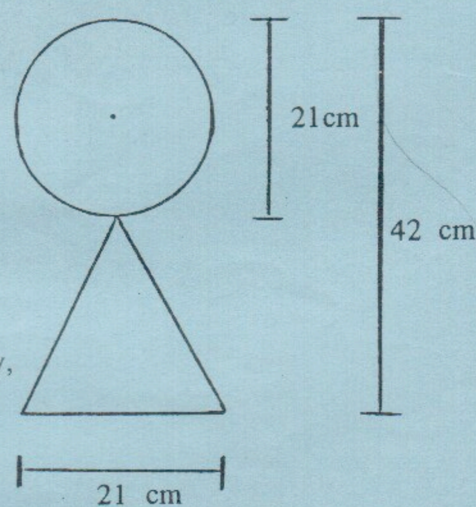
2. A team trophy for a football match is a cone supporting a sphere.

Calculate (taking $\pi = \frac{22}{7}$) the volume of

(i) the cone

(ii) the sphere

(iii) the smallest cylindrical box which will contain the trophy.



Each person on the team is to receive a replica of the trophy, each measurement of which is one seventh the original.

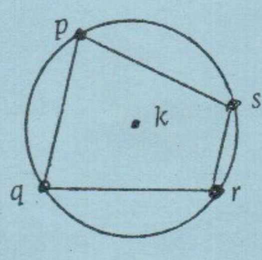
Calculate the volume of the smallest cylindrical box which will contain the replica.

Calculate the ratio $\dfrac{\text{Volume of team trophy box}}{\text{Volume of replica box}}$.

3. (i) Prove that the measure of the angle at the centre of a circle is twice the measure of an angle at the circle standing on the same arc.

(ii) p, q, r, s, are four points on a circle. Prove

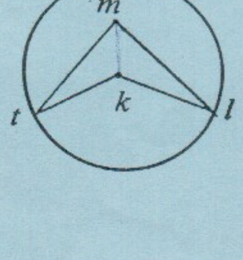
$|\angle qps| + |\angle qrs| = 180°$.



(iii) Points t and l are on a circle, centre k.

m is a point such that $|mk| < |kt|$.

Prove $|\angle tkl| < 2\,|\angle tml|$.



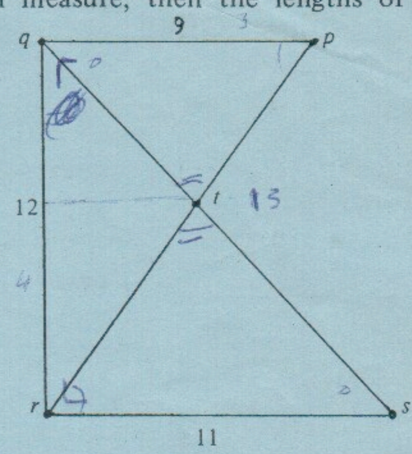
4. (i) Prove that if the angles of two triangles are equal in measure, then the lengths of their corresponding sides are proportional.

(ii) In the diagram, $pq \parallel rs$ and $pq \perp qr$.

The lengths of three of the lines are shown.

Calculate (iii) $|pr|$ 15

(iv) $|pt|$.



5. M is the line $2x - y = 3$.

Investigate if the points $q\,(2, 6)$ and $p\,(4, 5)$ are in M.

NO yes

Find the equation of the line through q

(i) parallel to M $2x - y = 2$

(ii) perpendicular to M. $\frac{1}{2}x + y = 7$

$9^2 + 12^2 =$

$(3(3))^2 + (3(4))^2 = (3(5))^2$

144
121
225

9×25

$\sqrt{225} = 15$

$y = 2x + 2$

$y = -\frac{1}{2}x + 2$

Calculate the area of the triangle pqr, if r is the intersection of the X-axis and the line through (2, 6) parallel to M.

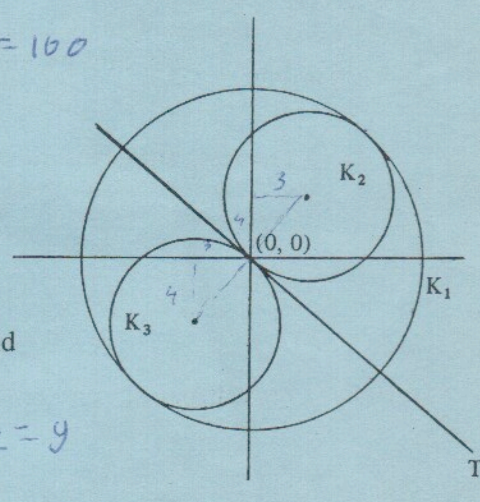
6. Write down the equation of the circle

$x^2 + y^2 = 100$

(i) K_1 centre (0, 0), radius length 10

(ii) K_2 centre (3, 4), when K_2 contains (0, 0)

(iii) K_3 centre $(-3, -4)$, when K_3 contains (0, 0).



Find (iv) the equation of the tangent T through (0, 0) and common to K_2 and K_3

$-\frac{3}{4}x = y$

$\frac{4x}{3} = y$

(v) the coordinates of points in $\mathbf{T} \cap \mathbf{K_1}$.

$x^2 + \frac{9}{16}x^2 = 100$

$(+8, -6)$

$(-8, +6)$

$16x^2 + 9x^2 = 1600$

$25x^2 = 25(64)$

OVER→

$x^2 = 64$

$x = \pm 8$ $y = \pm 6$

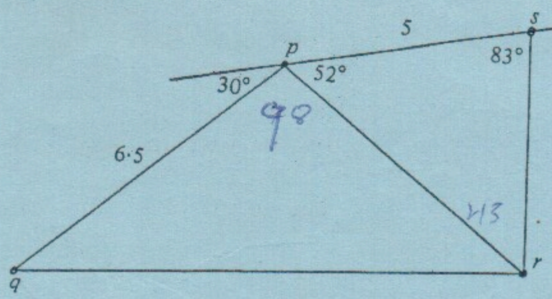
(35

7. (a) Using the data in the diagram calculate, correct to one place of decimals,

(i) $|pr|$

(ii) $|qr|$

[Use of scale-diagram not permitted]

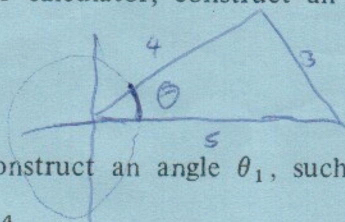


$$\frac{5}{\sin 45} = \frac{|pr|}{\sin 83}$$

$$|qr| = \sqrt{6.5^2 + |pr|^2 - 2(6.5)(|pr|)\cos}$$

(b) Without using Tables or calculator, construct an angle θ, such that,

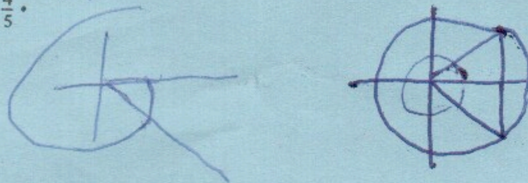
$$\cos\theta = \frac{4}{5}$$



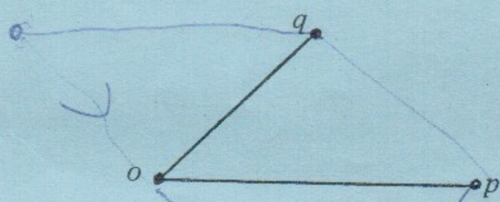
Hence, or otherwise, construct an angle θ_1, such that

$$\cos\theta_1 = -\frac{4}{5}.$$

S A
T C



8. (a) The diagram shows the origin o and two points p and q.

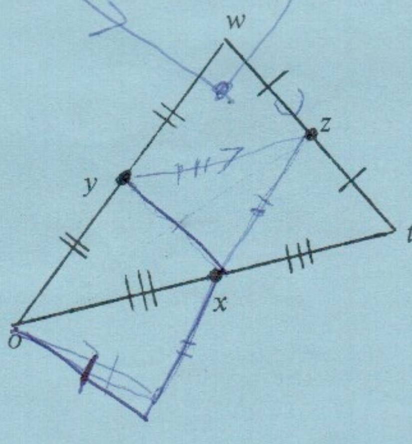


Copy the diagram and plot separately

(i) $\vec{k_1}$ equal to $\vec{p} - \vec{q}$

(ii) $\vec{k_2}$ equal to $\vec{q} - \vec{p}$.

(b) In the triangle otw, o is the origin and x, y, z are mid-points of the sides.



Express, in terms of $\vec{x}$ and $\vec{y}$

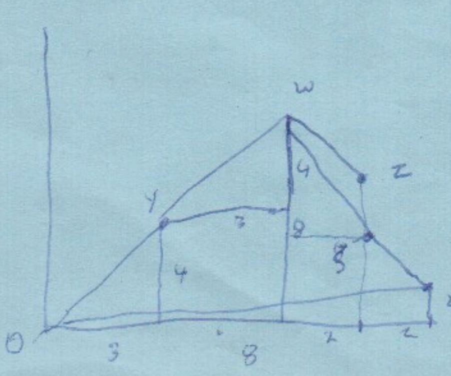
(i) $\vec{yz}$ $\quad\vec{x}$

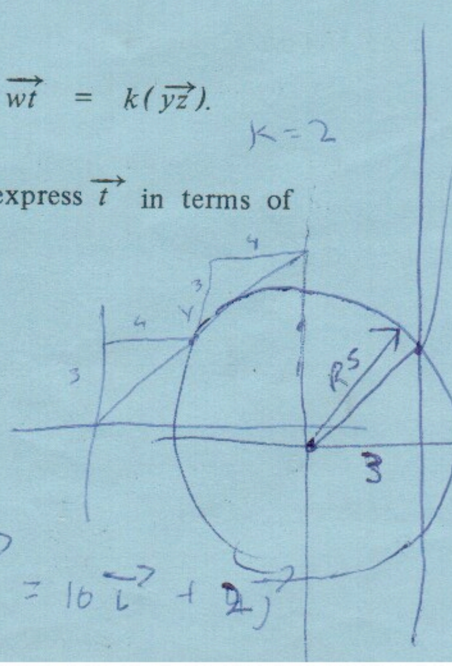
(ii) $\vec{wz}$. $\quad\vec{x} - \vec{y}$

Hence, or otherwise, find a scalar k such that $\vec{ow} + \vec{wt} = k(\vec{yz})$.

K=2

If then, $\vec{y} = 3\vec{i} + 4\vec{j}$ and $\vec{z} = 8\vec{i} + 5\vec{j}$, express $\vec{t}$ in terms of $\vec{i}$ and $\vec{j}$.





↑
W

$\vec{t} = 16\vec{i} + 2\vec{j}$