M.48

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LEAVING CERTIFICATE EXAMINATION, 1989

MATHEMATICS – ORDINARY LEVEL – PAPER II (300 marks)

FRIDAY, 9 JUNE – MORNING, 9.30 – 12.00

Attempt QUESTION 1 (100 marks) and FOUR other questions (50 marks each)

Marks may be lost if all your work is not clearly shown or if you do not indicate where a calculator has been used

1. (i) A person sells property for IR£80 000. The person pays a commission of 1% of the selling price. The person also pays tax on 45% of the selling price at the rate of 60%.
How much money has the person left after payment of commission and tax ?

(ii) Solve

$$3x^2 - 4(x + 2) - 7 = 0.$$

(iii) Factorize

$$(y^3 - 1) - (y - 1).$$

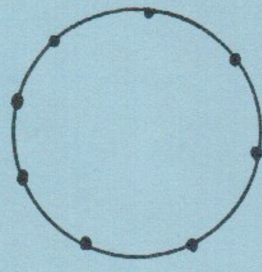
(iv) Solve the simultaneous equations

$$3x - y = 8; \qquad x = 1{\cdot}2 - 0{\cdot}4y.$$

(v) In a sequence, the general term $T_n = \frac{2^n}{3}$.

Show that $T_{n+1} \div T_n$ is a constant.

(vi) The diagram shows 8 points on a circle.
A chord is a line segment joining any two of these points.
Calculate the number of such chords that can be drawn.



(vii) On a graph, shade the set of points of the set

$$\{(x, y) \mid 2y \leqslant x, \quad x, y \in \mathbf{R}\}$$

and give a reason for your answer.

(viii) If $3^{x-1} = 243$, find the value of x.

(ix) The function f is defined as

$$f : \mathbf{R} \to \mathbf{R} : x \to x^2 - x.$$

Find the two values of k for which

$$f(k) = f(2k).$$

(x) Differentiate $\dfrac{x}{1 - x^3}$ with respect of x.

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2. Let $z = 2 - 3i$ and $w = 5 - 7i$, where $i = \sqrt{-1}$.

Plot z and w on an Argand diagram.

Verify that $|w - z| = 5$.

Draw the set K of all complex numbers such that each is a distance of 5 from z.

Give a geometrical meaning for $|w - z|$.

Investigate if $(-1 + i)$ is an element of K.

Write in the form $a + ib$ the image of w under the central symmetry in z.

3. A group of people form a club and over a period of time contribute to a fund to purchase equipment. The records showed the contributions as follows:

Contributions IR£	0 – 10	10 – 20	20 – 30	30 – 40	40 – 60
Number of club members	15	20	30	25	10

(Note: 0 – 10 means 0 is included but 10 is not etc.)

Calculate $\bar{x}$, the mean contribution per person, taking the contributions at the mid-interval values.

Construct the corresponding cumulative frequency table using the headings <10, <20, <30, <40 and <60 and draw the cumulative frequency curve.

Use the curve to estimate

the number of people in the range $\bar{x} - \sigma$ to $\bar{x} + \sigma$,

taking the standard deviation $\sigma = 13$.

4. If $f(x) = 2x^3 + 7x^2 + 2x - 3$, complete the following table:

x	$-3{\cdot}5$	-3	-2	-1	0	1
$f(x)$	-10					

Draw the graph of the function

$$f : x \to 2x^3 + 7x^2 + 2x - 3$$

in the domain $-3{\cdot}5 \leqslant x \leqslant 1$.

Use your graph to estimate

(i) the value of $f(-1{\cdot}5)$

(ii) the values of x for which the tangents to the graph have slope 0

(iii) the range of values of x in the given domain for which $x(2x^2 + 7x + 3) < 5 + x$.

5. (a) Solve

$$\frac{2}{x - 1} = \frac{x}{x^2 - 6}$$

(b) Evaluate

$$\frac{3! \; n!}{(n - 2)! \; 2n}$$

(c) Write out the first four terms of the expansion of

$$x^{12}(1 + \frac{2}{x})^6$$

in descending powers of x.

Find the sum of the first four terms when $x = 1$.

6. (a) The sum of the first five terms of an arithmetic series is 2 and the fifth term is $-\frac{6}{5}$.

Find the values of a and d. Hence, calculate the sum of the second five terms.

(b) A person put IR£W at compound interest in a Savings Bank. The rate for the first year was 10%. At the end of the first year the person put another IR£W in the Savings Bank. The rate for the second year was 8% and the rate for the third year was 5%. If the total investment at the end of the three years amounted to IR£5953·50, calculate W.

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7. A company assembles two models of microcomputer – model K and model T. The company must assemble at least 4 times as many of Model K as model T. If x of model K and y of model T are assembled, verify that $x \geqslant 4y$.

The assembly time for each model K is 18 hours and for each model T is 24 hours and a total of 192 hours is available in a given assembly period.

Graph the set showing the possible number of each model assembled in the period.

There is a ready market for each model and the company makes a profit of IR£150 on each model K and IR£100 on each model T. How many of each type should the company assemble to give a maximum profit ?

Indicate on your graph the region where the profit is greater than or equal to IR£600.

8. (a) Differentiate from first principles $2x - 3x^2$ with respect to x.

(b) (i) Find the slope and equation of the tangent to the curve

$$y = (3x - 2)(1 - x^2)$$

at the point $(0, -2)$.

(ii) If $y = (3x^2 - 5)^7$, show that $\dfrac{dy}{dx} > 0$

for all $x > 0,\ x \in \mathbf{R}$.

(c) Show that at $x = \frac{5}{3}$, the curve

$$y = x^3 - x^2 - 5x + 7$$

has a local minimum.

Find the co-ordinates of the local maximum.