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M48

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LEAVING CERTIFICATE EXAMINATION, 1988

MATHEMATICS – ORDINARY LEVEL – PAPER II (300 marks)

FRIDAY, 10 JUNE – MORNING, 9.30 – 12.00

Attempt QUESTION 1 (100 marks) and FOUR other questions (50 marks each)

Marks may be lost if all your work is not clearly shown or if you do not indicate where a calculator has been used

1. (i) A person has a salary of IR£10 000 per annum and tax free allowances of IR£2 000. Tax is deducted at the rate of 35% on $\frac{2}{5}$ of the taxable income and at the rate of 48% on the remainder. Find the amount of tax paid.

(ii) The length of a rectangle exceeds that of its width by 4 cm. The area of the rectangle is 320 cm^2.
Find the length of each side.

(iii) Solve

$$3(x + 1)^2 - 5x - 7 = 0.$$

(iv) Find k if

$$6x^3 - 17x^2 + 22x - 15 = (2x - 3)(3x^2 + kx + 5).$$

(v) Solve

$$\frac{x - 2}{3} = \frac{y + 5}{6}$$

$$5x = 7y.$$

(vi) What is the median wage of 10 students whose earnings in IR£ per week are as follows:

60, 180, 158, 84, 66, 54, 90, 96, 72, 48.

(vii) Find x when

$$\log_x 100\sqrt{10} = 5.$$

(viii) Graph the set

$$\{(x, y) \mid x \geqslant y, \quad x, y \in \mathbf{B}\}, \quad \mathbf{B} = \{1, 2, 3\}.$$

(ix) The function f is defined as

$$f : \mathbf{R} \to \mathbf{R} : x \to \frac{x+1}{2}.$$

Write the function f^2 (i.e., $f \circ f$) in the form

$$x \to ax + b.$$

(x) Let $y = x(x^2 - x + 1)$ for $x > 0$.

Find $\dfrac{dy}{dx}$.

Show that 2 is the value of $\dfrac{dy}{dx}$ at $y = x$.

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2. Let $z = 2 - 3i$, where $i = \sqrt{-1}$.

Show that z is a root of the equation

$$z^2 - 4z + 13 = 0.$$

Plot i, z and iz on an Argand diagram.

Verify that $|iz - z| > |iz| - |z|$.

If $z + i + 3(p + 2qi) = iz - 5$, find p and q for $p, q \in \mathbf{R}$.

3. A survey of 125 students gave the amount of money spent weekly in the canteen as follows:

Amount in IR£	0 – 10	10 – 15	15 – 20	20 – 25	25 – 30
Number of students	23	20	52	20	10

(Note: 10 – 15 means 10 is included but 15 is not, etc.)

Illustrate the above data by a histogram.
In which interval (class) does the median lie ?
Calculate the mean amount spent per student, taking the amounts at the mid-interval values.

The survey was repeated six months later. It was found that half the number of students in each interval, except the (0 – 10) interval, spent IR£5 less than they spent before.

Complete the new frequency distribution table:

Amount in IR£	0 – 10	10 – 15	15 – 20	20 – 25	25 – 30
Number of students					

4. If $f(x) = x^3 + x^2 + x - 2$, complete the following table:

x	-2	-1	0	1	2
$f(x)$					

Draw the graph of the function

$$f : x \rightarrow x^3 + x^2 + x - 2$$

in the domain $-2 \leqslant x \leqslant 2$, $x \in \mathbf{R}$.

Use your graph to find, as accurately as possible, the range of values of x for which $-5 \leqslant f(x) \leqslant 5$.

Using the same axes and the same scales, draw the graph of the function

$$g : x \rightarrow x^2 + x - 6$$

in the domain $-3 \leqslant x \leqslant 2$ for $x \in \mathbf{R}$.

Hence, use both graphs to estimate $\sqrt[3]{-4}$.

5. (a) Solve

$$\frac{4}{x+2} - \frac{2}{x-2} = 3 .$$

(b) Expand $(1 - 2x)^4$.

Use your expansion to find the value of x which satisfies

$$1 + 8x^2(3 + 2x^2) = 8x(1 + 4x^2) .$$

(c) A set of 10 pupils exchange handshakes once with each other. Calculate the number of handshakes between the 10 pupils.

6. (a) Write down T_n , the general term of an arithmetic sequence, in which $T_1 = a$ and the common difference is d.

In an arithmetic sequence the sixth term is 20 and the tenth term is four times the second term.

Find the values of a and d. Hence, calculate T_{101} .

(b) At the same time each year, for three consecutive years, a student borrowed IR£P at 12% per annum compound interest:

Year 1, student borrowed IR£P for 3 years at 12% per annum compound interest.
Year 2, student borrowed IR£P for 2 years at 12% per annum compound interest.
Year 3, student borrowed IR£P for 1 year at 12% per annum compound interest.

At the end of the three years the student repaid the amount due.

If the student repaid IR£8 400, calculate P .

7. A builder is to build 10 small shopping units on an 8000 m^2 site. These small shopping units are of two types – one will occupy an area of 500 m^2 and the other an area of 1000 m^2.

Graph the set showing the possible numbers of each type of small shopping unit that could be built.

The weekly rent from these two types of units is as follows:-

IR£125 for a 500 m^2 unit;
IR£200 for a 1000 m^2 unit.

How many of each type should be in the site to give maximum rent ?

Indicate on your graph the region where the rent would exceed IR£1000 per week.

8. (a) Differentiate from first principles

$$x^2 - 3x$$

with respect to x.

(b) (i) If $y = (2x - 3)^4$, find the value of x for which

$$\frac{dy}{dx} = 0.$$

(ii) Find the value of $\frac{dy}{dx}$ at $x = 0$ when

$$y = \frac{(2x - 3)^4}{4x - 1}.$$

(c) Find the co-ordinates of the local maximum of the curve

$$y = 3x^3 - x + 1.$$