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LEAVING CERTIFICATE EXAMINATION, 1987

MATHEMATICS – ORDINARY LEVEL – PAPER II (300 marks)

FRIDAY, 12 JUNE – MORNING, 9.30 – 12.00

Attempt **QUESTION 1** (100 marks) and **FOUR** other questions (50 marks each)

Marks may be lost if all your work is not clearly shown or if you do not indicate where a calculator has been used

1. (i) IR£600 is invested at $r\%$ per annum. The interest earned is subject to tax at 48%. After one year tax of IR£24·48 was paid. Calculate r.

(ii) Find the number of k if $x - 2$ is a factor of

$$x^3 + kx^2 - 10x + 8.$$

(iii) Factorise

$$(1 - x^3) - (1 - x^2)$$

(iv) In how many ways can the letters A, E, I, O, U be arranged if U can never be first ?

(v) Solve

$$x + y = 3; \quad y - z = 3; \quad z - x = -2.$$

(vi) 1, x, $2x$ are three numbers in geometric sequence. Find the value of $x \neq 0$.

(vii) If $4^{x+1} = 128$, find the value of x without using the tables.

(viii) On a graph, shade the set of points of the set

$$\{(x, y) \mid 3y \leqslant x, \quad x, y \in \mathbf{R}\}.$$

(ix) Let the function f be defined as

$$f : \mathbf{R} \longrightarrow \mathbf{R} : x \longrightarrow 2x^2 + 1.$$

If $f(x + 1) = f(x) + g(x)$, write $g(x)$ in the form $x \longrightarrow \ldots$

(x) Differentiate $(2x - 3x^2)^6$ with respect to x.

2. Let $z = -1 - 2i$ and $w = 1 + 2i$, where $i = \sqrt{-1}$.

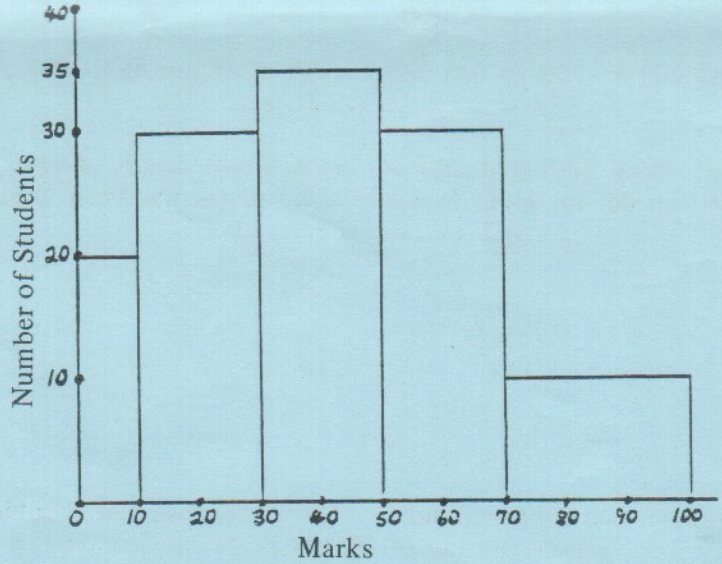
Write $2z - 3w$ and $\frac{z}{w}$ in the form $a + ib$.

Plot z and w on an Argand diagram.

The point t is the image of the point z^2 under the axial symmetry in the real axis (i.e. X-axis). Write t in the form $a + ib$.

The point t is also the image of the point w^2 under the axial symmetry in a line K. Name this line K.

3. (a)



Complete the frequency distribution table, below, for the histogram given in the diagram.

Marks	0 – 10	10 – 30	30 – 50	50 – 70	70 – 100
Number of Students		30			

(b) A garage owner recorded the amount of money spent by customers on petrol over a day. The results were:

Value of petrol sales IR£	0 – 5	5 – 10	10 – 15	15 – 20	20 – 30
Number of customers	50	150	400	300	100

(i) Taking the value of the sales at the mid-interval value, calculate the value of the petrol sales that day.

(ii) Complete the cumulative frequency table below:-

Value of petrol sales IR£	<5	<10	<15	<20	<30
Number of customers					

Draw the cumulative frequency curve.
Use this curve to estimate

the median sales value,
the number of customers who purchased more than IR£17 worth of petrol.

4. The function

$$f : x \to x^3 - 2x^2 - 6x + 4$$

is defined on the domain $-2 \leqslant x \leqslant 4$ for $x \in \mathbf{R}$.

Noting that $f(-2) = 0$ and $f(2) = -8$, draw the graph of the function.

Use your graph to find, as accurately as possible,

the range of values of $x > 0$ for which $f(x) < 0$.

Using the same axes and scales draw the graph of the function

$$g : x \to 2x - 5, \quad \text{for } x \in \mathbf{R}$$

Find

(i) the range of values of x for which $g(x) > f(x)$

(ii) the two roots of the equation

$$x^3 - 2x^2 - 8x + 9 = 0.$$

5. (a) Solve

$$\frac{1}{x} - \frac{3(x+1)}{x-1} = -1.$$

(b) Evaluate

$$6\binom{10}{4} - 5\binom{10}{5}$$

(c) When $\left(1 - \frac{x}{2y}\right)^{10}$ is expanded in ascending powers of x, find

(i) the coefficient of x^5 when $y = 1$

(ii) the sum of the first three terms when $x = 0{\cdot}1$ and $y = 0{\cdot}01$.

6. (a) In an arithmetic sequence the first term is 6 and the common difference is 6. Write down T_n, the general term of the sequence.

How many numbers are there between 1 and 1000 which are multiples of 6 ? Find the sum of these multiples.

(b) A person purchased an Investment Bond of IR£5000. At the end of the second year the bond had a value of IR£7 500. If the bond earned interest at the rate of 20% per annum during the first year, what was the rate of compound interest the bond earned for the second year ?

During the third year, the rate of compound interest which the bond earned was the average of the rates for the first two years. At the end of the third year, the person cashed the bond and in so doing paid 1% of the final value of the bond in commission. How much did the person receive and how much commission was paid ?

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7. A holiday campsite caters for caravans and tents. Each caravan accommodates 8 people and each tent accommodates 5 people. If there are x caravans and y tents on the site and if the site facilities cannot accommodate more than 400 people, write down an inequality to express this information.

Each caravan is allotted an area of 60 m^2 and each tent is allotted 50 m^2. The total area available for caravans and tents is $3\,600 \text{ m}^2$. Write down an inequality to express this information.

Graph the set showing the possible numbers of caravans and tents on the site.

If there were only caravans on the site, what is the maximum number of caravans which could be catered for ?

The charges on the site are IR£30 per caravan and IR£20 per tent.

How many caravans and how many tents should be on the site to give a maximum income ?

Indicate on your graph the region where the income would be less than IR£600.

8. (a) Differentiate from first principles

$$2x - x^2$$

with respect to x.

(b) (i) If $y = (1 + x)(1 - x)$, show that

$$\frac{dy}{dx} > 0 \quad \text{for all } x < 0, \quad x \in \mathbf{R}.$$

(ii) Show that the tangent to the curve

$$y = \frac{3x^2 + 2}{(1 + x)(1 - x)}$$

at the point $(0, 2)$ is parallel to the line $y = 0$.

(c) Find the coordinates of the local maximum and local minimum of the curve

$$y = x^3 - 3x + 7$$

and draw a rough graph of the curve.