MATHEMATICS – ORDINARY LEVEL – PAPER II (300 marks)

MONDAY, 16 JUNE – MORNING, 9.30 to 12.00

Attempt **QUESTION 1** (100 marks) and **FOUR** other questions (50 marks each)

Marks may be lost if all your work is not clearly shown
or if you do not indicate where a calculator has been used

(i)   IR£800 was invested at $r\%$ per annum. After one year the interest remaining when tax at 60% was deducted from it was IR£39·20. Calculate r.

(ii)   On a journey from Dublin to Berlin there were stops at London and Hanover lasting 1 hour 35 minutes and 55 minutes, respectively. The flight left Dublin at 10.15 a.m. Dublin time and reached Berlin at 4.40 p.m. Berlin time. If Berlin time is one hour ahead of Dublin time, calculate the actual flying time.

(iii)   Factorise as fully as possible:
$$(a - 2b)^2 - (2a - b)^2.$$

(iv)   Let $f(x) = 5x^3 - 7x + 5$. Find the remainder when $f(x)$ is divided by $(x + 1)$.

(v)   Solve
$$2(x - 1)^2 + x - 1 = 6.$$

(vi)   The general term of a sequence is given by
$$n(n^2 + 1).$$
Verify that the sequence is not geometric.

(vii) Let $t = \{(a, a), (a, b), (b, a), (b, b)\}$. Write out the set of couples of $t \circ t$.

(viii) By drawing a rough graph, or otherwise, find the set of values of $x \in \mathbf{R}$ for which
$$2 + 9x < 5x^2.$$

(ix)   Let the function f be defined as
$$f : \mathbf{R} \to \mathbf{R} : x \to 3(x - 1).$$
If 1 is an element of the codomain, find the corresponding element of the domain.

(x)   Differentiate $\dfrac{1}{x(1 - x)}$ with respect to x.

2. Verify that $5 - 2i$ is a solution of the equation

$$z^2 - 10z + 29 = 0.$$

Let $z_1 = 5 - 2i$ and $z_2 = 4 + i$.
Find z_3 such that $z_1 - z_3 = z_2$.

Plot the three complex numbers

$$z_1, \qquad iz_1, \qquad 8 - z_3$$

on the Argand diagram and verify that

$$|z_1 - 8 + z_3| = \sqrt{29}.$$

A circle is drawn on the Argand diagram having $8 - z_3$ as centre and $\sqrt{29}$ as radius. Show that this circle contains (passes through) iz_1.

3. A survey of workers in a factory gave the time spent travelling to work as follows:

TIME (minutes)	5 – 15	15 – 25	25 – 35	35 – 65
NO. OF WORKERS	5	30	50	20

(Note: 5 – 15 means 5 is included but 15 is not etc.)

Draw a histogram to illustrate the data.

Taking the time at the mid-interval value, calculate the mean time of travel per worker.

On a particular day a detour resulted in 20% of the workers in each class time, except the (35 – 65) class time, taking 10 minutes extra to travel. How many workers in the (35 – 65) class time did not come to work on that day if the mean travel time per worker increased by 1·7 minutes ?

4. If $f(x) = x^3 + 2x^2 - 7x - 3$, complete the following table:

x	-4	-3	-2	-1	0	1	2	$2\frac{1}{2}$
$f(x)$	-7	9		5	-3		-1	$7\frac{5}{8}$

Draw the graph of the function

$$f : x \rightarrow x^3 + 2x^2 - 7x - 3$$

in the domain $-4 \leqslant x \leqslant 2\frac{1}{2}$, $x \in \mathbf{R}$.

Use your graph to estimate

(i) the values of $x < 0$ for which $f(x) = 0$

(ii) the range of values of $x < 0$ for which

the tangents to the graph have positive slope.

Another function $g : x \rightarrow g(x)$ is such that a point of intersection of the graphs of f and g is used to estimate $\sqrt{7}$. Express g in the form $x \rightarrow$.
(Note: It is not necessary to draw the graph of g).

5. (a) Solve

$$x - y = 0$$
$$2x - 3(y - 1) = 0$$
$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2$$

(b) (i) The set of seven digits $\{2, 3, 4, 5, 6, 7, 8\}$ is used in writing down two digit products such as 2×3, 3×2, 5×8 etc. Assuming there are no products of the form 2×2, 3×3, etc., calculate the number of ways of *writing* the products.

(ii) Write down the middle term in the expansion of

$$(1 - 3x)^8$$

and find the value of this term when $x = \frac{1}{3}$.

6. (a) The first two terms of a series are

$$1 + 2 + \ldots$$

(i) If the series is geometric, write down the next two terms and show that S_{33}, the sum of the first 33 terms, is less than 2^{33}.

(ii) If the series is arithmetic, show that

$$S_n = \frac{n^2 + n}{2}$$

and find the sum of the first 100 terms.

(b) A person invested IR£1000 to be withdrawn completely in two equal instalments. The first instalment is to be withdrawn at the end of the second year. The second instalment is to be withdrawn at the end of the third year.

The investment earned compound interest at the rate of 15% per annum during the first year, at 12% per annum during the second year and at 10% per annum during the third year. Find the size of each instalment giving your answer correct to the nearest IR£.

7. A car hire firm supplies small and large cars. The firm must have at least twice as many small cars as large cars. The average daily costs on maintaining a small car are IR£15 and on maintaining a large car are IR£45. The total average daily overheads must not exceed IR£225.

Graph the set showing the possible numbers of each type of car which the firm can have.

If the daily hire charges are IR£45 for a small car and IR£60 for a large car, write down an expression for the daily profit and indicate by the letter K that region where the daily profit is greater than IR£225.

Also calculate the maximum daily profit that could be made.

8. (a) Differentiate from first principles
$$1 - 3x^2$$
with respect to x.

(b) (i) If $y = (2x - 1)(x^2 - 2)$, find the values of x for which
$$\frac{dy}{dx} = 0.$$

(ii) Find the slope of the tangent to the curve
$$y = (2x^2 + 3x - 1)^8$$
at the point $(0, 1)$.

(c) Find the coordinates of the local maximum of the curve
$$y = -x^3 + 3x - 5.$$