AN ROINN OIDEACHAIS M.47

LEAVING CERTIFICATE EXAMINATION, 1986

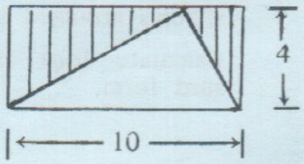
MATHEMATICS - ORDINARY LEVEL - PAPER I (300 marks)

THURSDAY, 12 JUNE - MORNING, 9.30 - 12.00

Attempt **Question 1** (100 marks) and **four** other questions (50 marks each)

Marks may be lost if all your work is not clearly shown or if you do not indicate where a calculator has been used

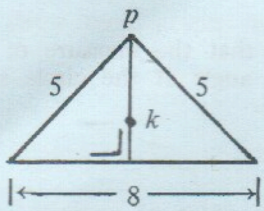
1. (i) Calculate the area of the shaded portion of the rectangle in the diagram.



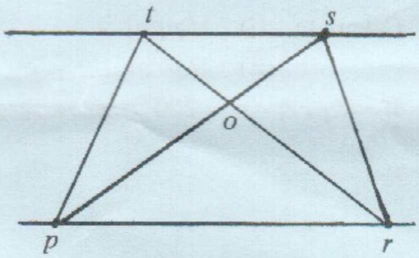
(ii) Express t in terms of x and y if

$$x - y = \frac{t + x}{t}.$$

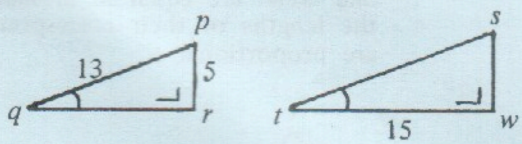
(iii) The lengths of the sides of a triangle are 5, 5 and 8. The medians intersect in k. Calculate $|pk|$.



(iv) In the diagram, $ts \parallel pr$. Prove that areas of the two triangles tpo, rso are equal.



(v) pqr and stw are two triangles, $|\angle prq| = |\angle swt| = 90°$, and $|\angle pqr| = |\angle stw|$. Calculate $|ts|$.



(vi) If $p = (6, 7)$, $q = (7, 6)$, calculate $|pq|$.

(vii) The equation of a circle is

$$x^2 - 6x + 9 + y^2 - 10y + 25 = 1.$$

Find the coordinates of its centre.

(viii) Find the equation of the image of the line $x - y = 0$, under the translation

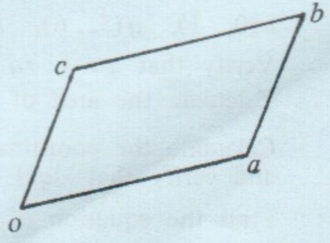
$$(0, 0) \rightarrow (0, -1).$$

(ix) $\sin(x + 4\pi) = 0{\cdot}75$. Find the value of $\sin x$.

(x) $oabc$ is a parallelogram, where o is the origin.

$$\vec{a} = k\vec{i} + \vec{j}$$
$$\vec{b} = 6\vec{i} + t\vec{j}$$
$$\vec{c} = 2\vec{i} + 5\vec{j}$$

Find the value of k and the value of t.



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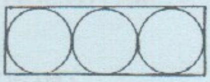
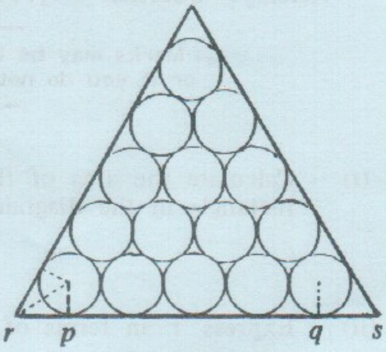
2. (i) Calculate, in terms of π, the volume of a sphere of radius 3 cm.

Three such identical spheres fit exactly into a closed cylindrical container. Calculate the internal volume of the container correct to the nearest cm^3 taking π to be 3·14.



(ii) An equiangular triangular frame holds 15 such spheres arranged as in the diagram.

Calculate $|pq|$ and $|rs|$, leaving the latter in surd form.



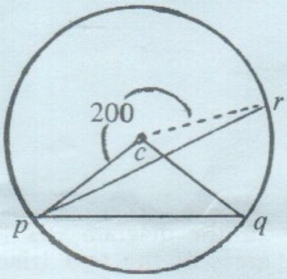
3. Prove that the measure of the angle at the centre of a circle is twice the measure of an angle at the circle standing on the same arc.

r is a point of a circle, centre c, $|\angle cpq| = 40°$.

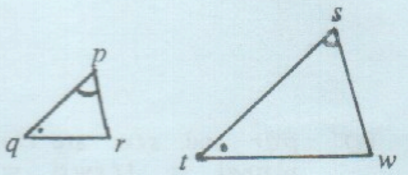
Calculate (i) $|\angle pcq|$

(ii) $|\angle prq|$.

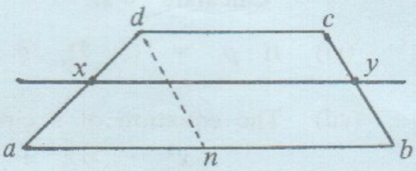
If $|\angle pcr| = 200°$, see diagram, calculate $|\angle crp|$.



4. (i) If the angles of the two triangles pqr and stw are equal in measure, prove the lengths of their corresponding sides are proportional.

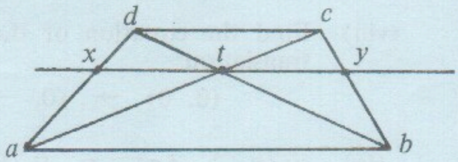


(ii) $abcd$ is a quadrilateral, $ab \parallel dc$.
A line xy is drawn parallel to ab.
Prove $|xa| : |ad| = |yb| : |bc|$.
[a construction line $dn \parallel cb$ is shown.]



(iii) $abcd$ is the quadrilateral of (ii).
The diagonals $[ac]$, $[db]$ intersect in t, through which a line $xy \parallel ab$ is drawn.
Prove

$$|xt| = |ty|.$$



5. $p(0, 1)$, $q(5, 6)$, $r(2, 7)$ are three points.
Verify that $pr \perp rq$.
Calculate the area of the triangle pqr.

Calculate the coordinates of w, the image of q under the central symmetry in r, and verify that $|qr| = |rw|$.

Find the equation of pw.

6. K_1 is the circle $x^2 + y^2 = 9$.

(i) Write down the length of the radius of K_1.

(ii) Calculate the coordinates of the points common to K_1 and the line $x - 2y + 3 = 0$.

Write down the equation(s) of

(iii) K_2, the image of K_1 under the translation $(1, 0) \rightarrow (10, 0)$.

(iv) K_3 and K_4, two of the circles which touch each of K_1 and K_2 and which have their centres on the x-axis.

7. (a) In the triangle pqr, $|pq| = 8$, $|qr| = 4$, $|rp| = 5$.
Calculate $|\angle qrp|$ and round off to the nearest degree.

(b) Using the same axes and scales, sketch the graph of

(i) $x \rightarrow \sin x$
(ii) $x \rightarrow \cos x$,

in the domain $0 \leqslant x \leqslant 2\pi$, $x \in \mathbf{R}$.

Assuming 2π is the period of both $\sin x$ and $\cos x$, show their graphs in $-2\pi \leqslant x \leqslant 0$.

Estimate

(iii) $\sin(\frac{\pi}{4} - 2\pi)$

(iv) $\cos(\frac{\pi}{4} - 2\pi)$

(v) $\sin(\frac{\pi}{2} - 2\pi)$.

8. (a) o, p, q are points, o is the origin.
Using separate diagrams, show points k_1, k_2, such that

(i) $\overrightarrow{ok_1} = \vec{p} + \vec{q}$

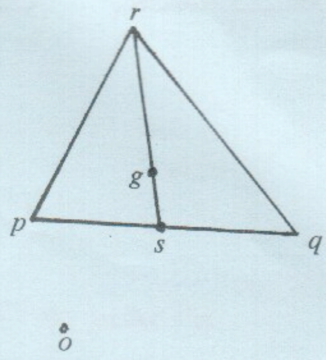
(ii) $\overrightarrow{ok_2} = \vec{p} - \vec{q}$

(b) pqr is a triangle and o is the origin.
$|ps| = |sq|$. Express

(i) $\overrightarrow{os}$ in terms of $\vec{p}$ and $\vec{q}$

(ii) $\overrightarrow{rs}$ in terms of $\vec{p}$, $\vec{q}$ and $\vec{r}$

(iii) $\overrightarrow{og}$ in terms of $\vec{p}$, $\vec{q}$ and $\vec{r}$, given that $|sg| = \frac{1}{3}|sr|$.



If $\vec{p} = -2\vec{i} + 3\vec{j}$
$\vec{q} = 7\vec{i} + 3\vec{j}$
$\vec{r} = \vec{i} + 9\vec{j}$

express $\vec{g}$ in terms of $\vec{i}$ and $\vec{j}$.