LEAVING CERTIFICATE EXAMINATION, 1985

MATHEMATICS – ORDINARY LEVEL – PAPER II (300 marks)

MONDAY, 17 JUNE – MORNING, 9.30 to 12.00

Attempt **QUESTION 1** (100 marks) and **FOUR** other questions (50 marks each)

Marks may be lost if all your work is not clearly shown

1. (i) When the exchange rate is IR£1 = 10 Kroner a person changes IR£150 into Kroner. A charge is made for this service. What is this percentage charge if the person gets 1470 Kroner ?

    (ii) Express

    $$\frac{1\cdot4 \times 10^3 + 5\cdot6 \times 10^2}{7 \times 10^{-1}}$$

    in the form $a \cdot 10^n$, where $1 \leqslant a < 10$ and $n \in \mathbf{Z}$.

    (iii) Factorise:

    $$(x + 2y)^2 + x^2 - 4y^2.$$

    (iv) If $x = 2$ is a root of

    $$2x^3 + x^2 - 13x + 6 = 0$$

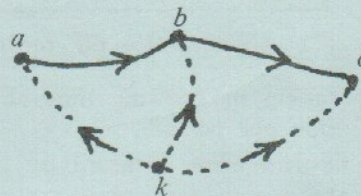   find the other two roots.

    (v) Express the product

    $$4^{\frac{1}{2}} \cdot 8^{-\frac{2}{3}}$$

    in the form 2^k.

    (vi) $f = \{(a, b), (b, c), (\ ,\ )\}$

    $g = \{(k, a), (k, b), (k, c)\}$

    If $f \circ g = g$, what other couple should be be included in f ?

    (vii) The lengths of the three sides of a right angled triangle are in arithmetic sequence. Find one set of suitable lengths.

    (viii) Calculate the standard deviation of the set

    $$\{-1,\ 0,\ 2,\ 3\}.$$

    (ix) The function f is defined as:

    $$f : \mathbf{R} \to \mathbf{R} : x \to 3x - 7.$$

    If $f(2) = k\,f(-2)$, find k.

    (x) Find the coordinates of the point on the curve

    $$y = x^2 - x - 6$$

    at which the tangent to the curve has a slope of -1.

2. Let $z_1 = 2 - 3i$ and $z_2 = -1 + 4i$, where $i = \sqrt{-1}$.

Plot z_1 and z_2 on the Argand diagram and prove that z_1 is nearer than z_2 to the origin.

Verify that $\dfrac{|z_1|}{|z_2|} = \left|\dfrac{z_1}{z_2}\right|$.

z_3 is the image of z_2 under the central symmetry in the origin. Express z_3 in the form $a + bi$ and plot it.

If $z_1 - \alpha(z_3 - z_1) = k$, where $\alpha, k \in \mathbf{R}$, find α and k.

3. (a) Two supermarkets, S_1 and S_2, made reductions in the prices of the four items A, B, C, D as shown in the table:

	A	B	C	D
S_1	10p	3p	5p	20p
S_2	8p	4p	3p	21p

Which supermarket had the best average reduction per item ?

If the sales of the four items A, B, C, D in each supermarket were in the ratio $2 : 4 : 1 : 3$, respectively, which, now, had the best average reduction ?

(b) The following table gives the number of people who watched television for a period of time on a certain day:

Viewing Time (minutes)	0 – 60	60 – 120	120 – 180	180 – 240
No. of People	130	480	300	90

(Note: 0 – 60 means 0 is included but 60 is not etc.)

Taking the viewing time at the mid-interval value, calculate the mean viewing time per person.

Explain what is meant by the "median" viewing time.

Use a cumulative frequency curve (based on $< 60, < 120, < 180, < 240$) to estimate the median viewing time.

4. The function

$$f : x \rightarrow 2x^3 - 5x^2 - 2x + 5$$

is defined on the domain $-2 \leqslant x \leqslant 3$ for $x \in \mathbf{R}$.

Verify that

$$f(-2) = -27 \quad \text{and} \quad f(-1) = 0$$

and draw the graph of the function.

Use your graph to estimate

(i) the value of $f(1{\cdot}75)$

(ii) the roots of the equation $f(x) + 20 = 0$

(iii) the set of values of x for which $2x^3 \geqslant 5x^2 + 2x$.

5. (a) Solve the simultaneous equations:

$$\frac{1}{x} = 2$$

$$y(3 - 2x) - 6 = 0$$

$$4x - y - 3z = 1.$$

(b) Show that $\binom{7}{3} = \binom{7}{4}$.

When $(1 - 5x)^7$ is expanded, find

    (i)   the coefficient of x^3

    (ii) the value of 5th term when $x = \frac{1}{5}$.

6. (a) The sum, S_n, of the first n terms of an arithmetic series is given by

$$S_n = 3n^2 - 6n.$$

Calculate S_2 and S_1 and find T_2, the 2nd term of the series.

If $T_n = a + (n - 1)d$, find the value of "a" and the value of "d" and say which term of the series is 333.

(b) A person invested IR£1000 at 15% per annum. At the end of the first year the person withdrew IR£500.

During the 2nd year the rate of interest was 12% per annum. At the end of the 2nd year the person withdrew a certain amount.

During the 3rd year the interest was 10% per annum. At the end of the 3rd year the person's investment was then worth 55% of the initial investment.

How much was withdrawn at the end of the 2nd year ?

7. A ship has space for at most 200 containers which are of two types —
    refrigerated and unrefrigerated. Each refrigerated container carries a load of
    3 tonnes and each unrefrigerated container carries a load of 8 tonnes. The
    maximum load the ship can carry is 1200 tonnes. Freight charges on each
    refrigerated container are IR£100 and on each unrefrigerated container IR£80.

    Graph the set showing the possible numbers of each type of container that the
    ship can carry.

    If operating costs on each journey amount to IR£14 000, indicate by the letter
    K that region of your graph where the ship is not operating at a loss.

    Calculate the maximum profit if a ready supply of each container is available.

8. (a) Differentiate from first principles

$$x - x^2$$

with respect to x.

(b) (i) If $y = \dfrac{x}{1 - x^2}$, show that $\dfrac{dy}{dx} > 0$ for all $x \in \mathbf{R}$.

(ii) Find the value of $\dfrac{dy}{dx}$ at $x = -1$ when

$$y = (2x^2 - 3)^7.$$

(c) Draw a rough graph of a function $y = f(x)$ which satisfies the following
conditions at the same time

$$y = 0 \quad \text{at} \quad x = 0$$

$$\frac{dy}{dx} = 0 \quad \text{at} \quad x = 0$$

$$\frac{dy}{dx} = 1 \quad \text{at} \quad x = 1$$

$$\frac{dy}{dx} = 0 \quad \text{at} \quad x = 2$$

$$\frac{dy}{dx} > 0 \quad \text{for} \quad 0 < x < 2.$$