LEAVING CERTIFICATE EXAMINATION, 1985

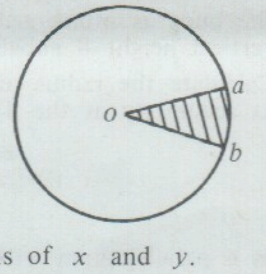
MATHEMATICS - ORDINARY LEVEL - PAPER 1 (300 marks)

THURSDAY, 13 JUNE - MORNING, 9.30 - 12.00

Attempt **Question 1** (100 marks) and **four** other questions (50 marks each)

Marks may be lost if all your work is not clearly shown.

1. (i) Find the area of the shaded portion of the disc, centre o, if $|\angle aob| = 36°$, $|ob| = 10$. Take π to be $3{\cdot}14$.



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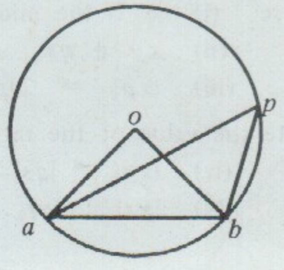
(ii) If $\frac{1}{f} - \frac{1}{x} = \frac{1}{y} - \frac{1}{f}$, express f in terms of x and y.

$\frac{2xy}{x+y}$

(iii) The length of each side of a triangle is l. Express the area of the triangle in terms of l.

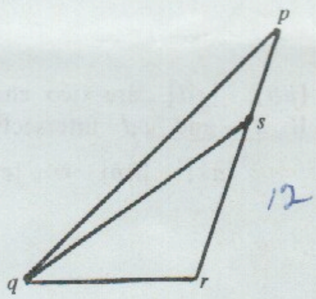
$l^2 \frac{\sqrt{3}}{4}$

(iv) p is a point of a circle, centre o, $|\angle oab| = 40°$. Calculate $|\angle apb|$.



50°

(v) If area Δpqr : area $\Delta pqs = 3 : 1$, and $|ps| = 6$, find $|sr|$.



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(vi) Find the value of k, if the line

$$3x + ky - 22 = 0$$

contains the point $(2, -4)$.

−4

(vii) Find the coordinates of the centre of the circle

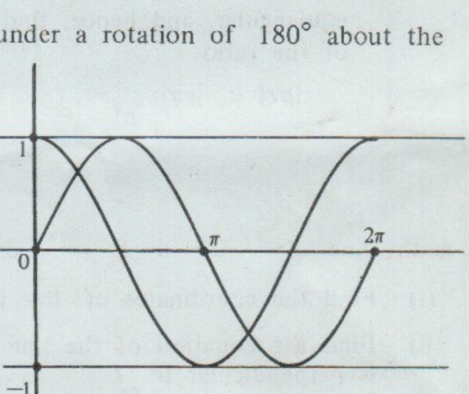
$$(x - 1)^2 + y^2 + 4y = 0.$$

(1, −2)

(viii) $(6, -1)$ is the image of the point (p, q) under a rotation of $180°$ about the origin. Find p and q.

(−6, 1)

(ix) Graphs of $\sin x$ and $\cos x$ in $0 \leqslant x \leqslant 2\pi$ are shown. State two values of x in $2\pi \leqslant x \leqslant 4\pi$, such that $\sin x = \cos x$.



(x) If $(3\vec{i} + 4\vec{j}) - (a\vec{i} - b\vec{j}) = -3\vec{j}$, find the value of a and the value of b.

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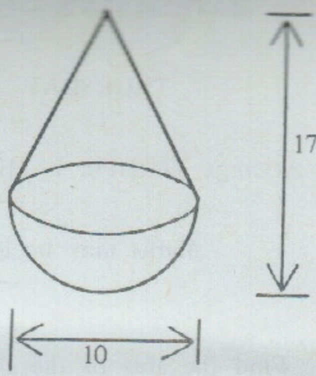
2. A buoy consists of a cone standing on a hemisphere. The overall height of the buoy is 17. The diameter of the base of the cone is 10.

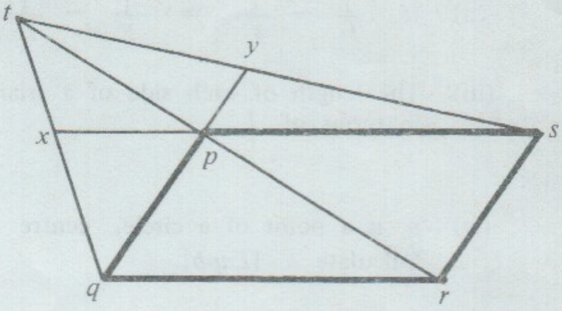


(i) Calculate, in terms of π, the volume of the buoy.

(ii) While floating vertically, $\frac{9}{11}$ of the volume of the buoy is submerged, leaving a cone of vertical height 8 above water.

Calculate the radius of the base of this cone, as accurately as the Tables allow.

3. $pqrs$ is a parallelogram with $|qr| = 2|qp|$. t is a point on rp, such that $|tp| = |pr|$. sp intersects tq in x and qp intersects ts in y.



Prove (i) x is the mid-point of $[tq]$

(ii) $xy \parallel qs$

(iii) $|xp| = 2|py|$

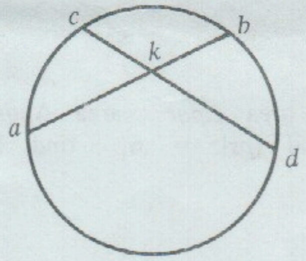
State the value of the ratios:

(iv) $|xy| : |qs|$

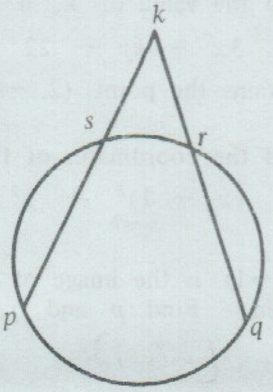
(v) $|xs| : |qr|$.

4. (a) $[ab]$, $[cd]$ are two chords of a circle. If ab and cd intersect in k, prove

$$|ak| . |kb| = |ck| . |kd|.$$



(b) $pqrs$ are points of a circle. ps, qr intersect in k.



(i) If $|ks| = 2$, $|kp| = 6$, $|kr| = 3$ calculate $|kq|$.

(ii) Prove the triangles kpr, ksq are equiangular, and hence, find the value of the ratio

$$|pr| : |sq|.$$

5. L is the line $x - 2y + 5 = 0$.

(i) Find the coordinates of the point, r, where L intersects the y-axis.

(ii) Find the equation of the line M, which contains the point $p(\frac{5}{2}, 0)$ and is perpendicular to L.

Calculate

(iii) the coordinates of q, if $L \cap M = \{q\}$.

(iv) the area of the quadrilateral, $opqr$, if o is the origin.

6. C is the circle $x^2 + y^2 = \frac{169}{9}$.

Write down (i) the length of the radius of C

(ii) the coordinates of the points in which C cuts the axes.

K is the image of C under the translation $(1, 1) \rightarrow (\frac{13}{3}, -7)$.

(iii) write down the equation of K

(iv) verify that the point $p(\frac{5}{3}, -4)$ is common to both C and K

(v) verify that p is the only point common to both C and K.

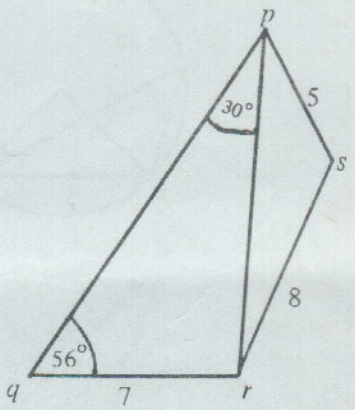
7. (a) If A, B, C denote the three angles of a triangle and a, b, c denote the lengths of the sides opposite these angles, prove

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

(b) $pqrs$ is a quadrilateral.

Using the data in the diagram find $|pr|$ correct to the nearest integer.

Use this value of $|pr|$ to calculate $|\angle psr|$, as accurately as the Tables allow.



8. oab is a triangle, o is the origin.
p, q are points on $[oa]$ and $[ab]$, respectively, such that

$$|ap| : |po| = |bq| : |qa| = 1 : 2.$$

Express (i) $\vec{bq}$

(ii) $\vec{oq}$

(iii) $\vec{qp}$, in terms of $\vec{a}$ and $\vec{b}$.

If $\vec{a} = 6\vec{i} - 3\vec{j}$ and $\vec{b} = 3\vec{i} + 9\vec{j}$ find $\vec{qp}$ in terms of $\vec{i}$ and $\vec{j}$.

Calculate $|\vec{qp}|$.