LEAVING CERTIFICATE EXAMINATION, 1984

MATHEMATICS – ORDINARY LEVEL – PAPER II (300 marks)

WEDNESDAY, 13 JUNE – MORNING, 9.30 to 12.00

Attempt **QUESTION 1** (100 marks) and **FOUR** other questions (50 marks each)

Marks may be lost if all your work is not clearly shown

1. (i) On paying a bill a customer was allowed a discount of 10%. He paid IR£180. How much would he have paid if he had been allowed a discount of 5% ?

(ii) Write the ratio $\frac{1}{2} : \frac{1}{5}$ in the form $p : q$ where $p, q \in \mathbf{Z}$.

(iii) Solve the equation $x - \dfrac{6}{x-1} = 2$.

(iv) Factorise $8x^3 + 27y^3$.

(v) If $3^{2x+1} = 81$, find x without the use of the Tables.

(vi) Find the sum of the first 100 terms of the arithmetic series

$13 + 10 + 7 + \ldots\ldots\ldots\ldots$

(vii) Calculate $\dfrac{10!}{6!\ 4!}$.

(viii) $f = \{(a, b), (b, c), (c, k)\}$

$g = \{(a, b), (b, d), (d, k)\}$

Write out the couples, if any, of $f \circ g$.

(ix) Graph the set A defined by

$A = \{(x, y) \mid 3x + 4y \geqslant 24, \ x, y \in \mathbf{R}\}$

and write A clearly on the set.

(x) Find the value of $\dfrac{dy}{dx}$ at $x = -1$ when

$y = (1 - x)(1 - x - x^2)$.

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2. Verify that the complex number $z_1 = 3 - 2i$ is a root of the equation

$$z^2 - 6z + 13 = 0$$

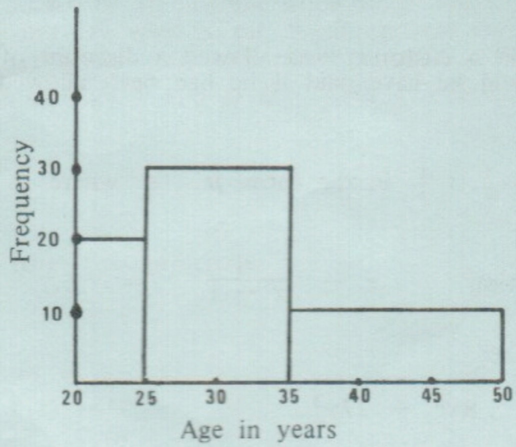
and find z_2, the other root of the equation.

On an Argand diagram plot the complex numbers z_1 and z_2.

z_3 is the image of z_1 under the central symmetry in z_2. Express z_3 in the form $a + ib$ and plot it on the Argand diagram.

Investigate if $|z_1 - z_2| = |z_1| - |z_2|$.

3. (a) The distribution of the ages of people attending a meeting is shown in the histogram



If there were 30 people in the 25 – 35 year age group, how many people were at the meeting ?

(b) 100 pupils were given a problem to solve. The following grouped frequency distribution table gives the numbers of pupils who solved the problem in the given time interval :

Time (minutes)	0 - 10	10 - 30	30 - 50	50 - 100
Frequency	10	21	47	22

(Note: 0 - 10 means 0 is included but 10 is not etc.)

Verify, using the mid-interval time values, that the average time taken per pupil to solve the problem is 40 minutes.

Assuming that the standard deviation, σ, is 22 minutes, use a cumulative frequency curve to estimate the percentage of pupils who solved the problem in the time interval $[40 - \sigma, 40 + \sigma]$ minutes.

4. The function

$$f: x \to x^3 - 4x^2 + 4$$

is defined for $-2 \leqslant x \leqslant 4,\ x \in \mathbf{R}$.

Draw the graph of f.

Find from your graph, as accurately as you can, the values of x for which

(i) $f(x) = 2$

(ii) $x^3 - 4x^2 - x + 4 = 0$.

Find, also, the range of values of h for which

$$f(x) - h = 0.$$

has one root only.

5. (a) The nth term of a sequence is given by

$$T_n = 2n + 1.$$

Write down an expression for the $(n - 1)$th term and hence deduce that the sequence is arithmetic.

Show that the sum of the first n terms is given by

$$S_n = n^2 + 2n.$$

Calculate the average (mean) of the first 51 terms of the sequence.

(b) Two people, A and B, invested IR£5000 each for one year. A invested at 2% per month, compound interest, while B invested at 24% per annum.

Calculate by how much is the income of A greater than the income of B.

[Note: You may take $(1{\cdot}02)^n$ as $1 + (0{\cdot}02)n$]

6. (a) Solve the simultaneous equations

$$2x - y = 1$$

$$xy = 6.$$

(b) Write out the first three terms of the expansion of

$$(1 - 3x)^6$$

in ascending powers of x.

Use your result to evaluate

$$(0{\cdot}997)^6$$

correct to four places of decimals.

7. A manufacturer produces two products P and Q. The time in hours required for the Cutting and the Finishing of each unit produced is shown in the table:

	P	Q
Cutting hours per unit	6	6
Finishing hours per unit	3	6

There are at most 180 hours available for Cutting and at most 150 hours for Finishing.

Assuming that a profit of IR£50 is made on the sale of each unit of P and a profit of IR£10 on the sale of each unit of Q and that there is a ready sale for both products:

(i) Graph the set of all possible sales of P and Q.

(ii) Graph the set of all possible sales of P and Q that yield a profit of IR£700 and say what is the maximum sale for P and the maximum sale for Q that would yield this profit.

(iii) Find the sales of P and Q that yield a maximum profit.

8. (a) Differentiate from first principles

$$1 - x^3$$

with respect to x.

(b) Find the value of $\frac{dy}{dx}$ at $x = 2$ when

$$y = \frac{x^3 - 4x}{x^2 - 1} .$$

(c) Show that the x-axis is a tangent to the graph of

$$y = (x^3 - 4x)^5$$

at $x = 2$.

(d) The boundary of a rectangular field is 100 m in length. If one side of the field is of length x metres, show that the area, y, of the field is given by

$$y = x(50 - x) .$$

Hence show that the maximum area of a rectangular field having a boundary of length 100 m is 625 m^2.