LEAVING CERTIFICATE EXAMINATION, 1984

MATHEMATICS - ORDINARY LEVEL - PAPER I (300 marks)

FRIDAY, 8 JUNE - MORNING, 9.45 - 12.15

Attempt **Question 1** (100 marks) and **four** other questions (50 marks each)

Marks may be lost if all your work is not clearly shown.

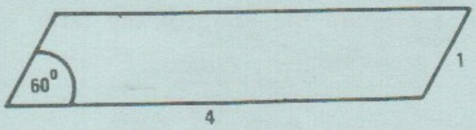
1. (i) 1010 pages of a telephone directory together have a thickness of 3·5 cm. Calculate the thickness of a single page in cm correct to three decimal places.

(ii) Express q in terms of p, h, k when

$$\frac{q}{p+q} = \frac{h+k}{h}.$$

(iii) If IR£1 is worth 78p sterling, find the IR£ value of £39 sterling.

(iv) Calculate the area of the parallelogram.

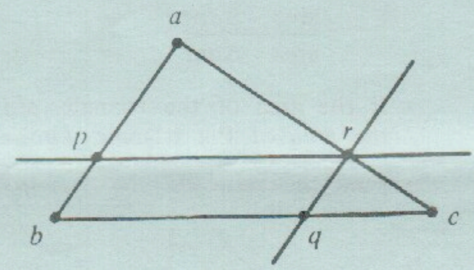


(v) In the Δabc, $pr \parallel bc$, $rq \parallel ab$.
If $|ap| : |pb| = 2 : 1$, state the value of each of the following ratios

$|ar| : |rc|$

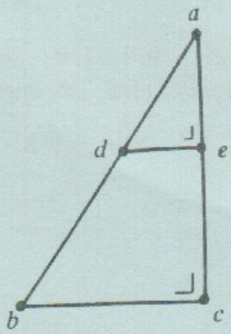
and

$|bc| : |qc|$.



(vi) abc, ade are right angled triangles as shown.
Prove

$$\frac{\text{area } \Delta ade}{\text{area } \Delta abc} = \frac{|de|^2}{|bc|^2}$$

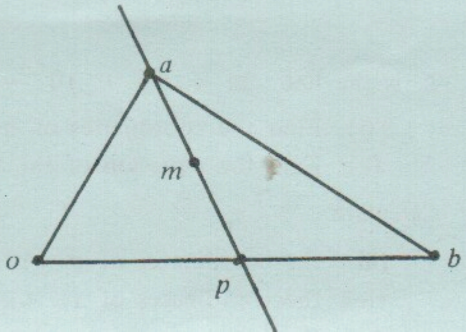


(vii) Calculate the length of the radius of a circle if one of its diameters has end points $(3, -1)$ and $(-3, 1)$.

(viii) Find the coordinates of the image of $(-3, 4)$ under the rotation about the origin of $+90°$.

(ix) If $13 \sin x = 5$ where $0 \leqslant x \leqslant \frac{\pi}{2}$, find the value of $\sin 2x$ without reading the values from the Tables.

(x) ap is a median of the Δaob.
m is the midpoint of $[ap]$.
Express $\vec{m}$ in terms of $\vec{a}$ and $\vec{b}$, where o is the origin.



2. The lower portion (A) of a test-tube is hemispherical and the upper portion (B) is cylindrical.

The length of the test–tube is 14·5 cm and its diameter is 3 cm.
Calculate

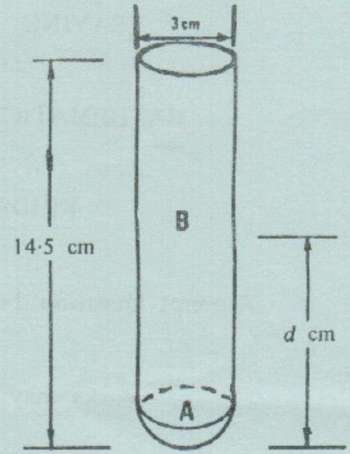
(i) the length of **B**

(ii) the volume of **B**, in terms of π

(iii) the volume of the test-tube (i.e. the volume of **A** and **B**) in terms of π.

If water is poured into the test-tube find

(iv) the depth, d, of the water when its volume is half the volume of the test-tube.



3. (i) Prove that the areas of two triangles of equal height are proportional to the lengths of their bases.

(ii) $abcd$ is a parallelogram. The diagonals intersect in k. t is the mid point of $[ab]$.

State the value of the ratio

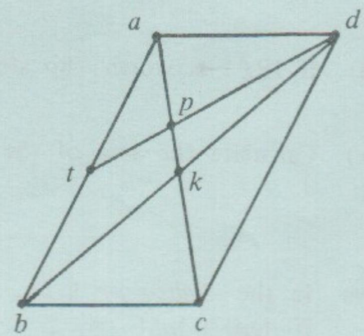
$$\frac{\text{area } \Delta dkc}{\text{area } \Delta dac}$$

and

$$\frac{\text{area } \Delta acd}{\text{area } \Delta atd}.$$

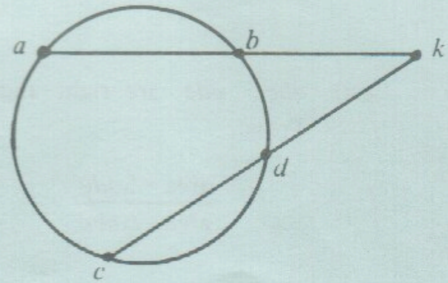
If the area of the triangle atd is 6, state the area of the triangle kbc.

Find the value of $|ap| : |pk|$.



4. (i) $[ab]$ and $[cd]$ are two chords of a circle. If ab and cd intersect in k, prove

$$|ka| \, . \, |kb| = |kc| \, . \, |kd|$$

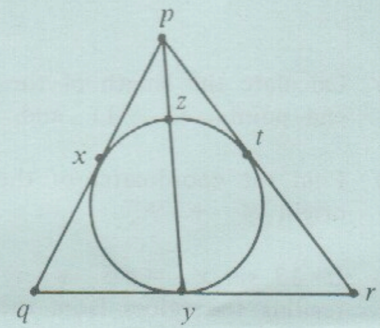


(ii) pqr is a triangle and xyt is its incircle. Prove

$$|px|^2 = |pt|^2 .$$

Hence, prove

$$|pr| + |rq| > |pq| .$$



5. L is the line $2x - 5y + 10 = 0$.

(i) Find the coordinates of q, i.e. where L intersects the x-axis.

(ii) Find the equation of the line M through $r(2, 0)$, the slope of M being $\frac{5}{2}$.

Calculate

(iii) the coordinates of p where p is $L \cap M$

(iv) the coordinates of s, the fourth point of the parallelogram $pqsr$

(v) the area of $pqsr$.

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6. (i) Write down the equation of the circle, S, of radius length $\sqrt{13}$, centre the origin.

(ii) Calculate the coordinates of the points of intersection of S and the line $2x + 3y = 0$.

(iii) Find the equation of T, the tangent to S at the point $(3, -2)$ of the circle.

(iv) K is the image of S under the axial symmetry in the tangent T. Write down the equation of K.

(v) p is a point of K which is farther from the x-axis than any other point of K. Find the y-coordinate of p in the form $a + \sqrt{b}$.

7. (a) Using the same axes and scales draw the graph of

(i) $\sin x$

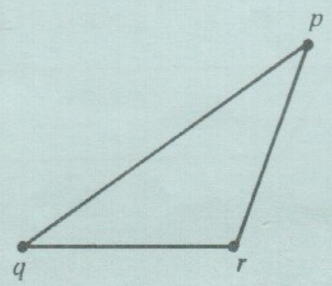
(ii) $\sin \frac{x}{2}$

in the domain $-2\pi \leqslant x \leqslant 2\pi$.

Given that the period of $\sin \frac{x}{2}$ is 4π, state a value of x, greater than 2π, at which the graphs, if continued, would intersect.

(b) In the triangle pqr, $|pq| = 16$, $|qr| = 10$ and $|\angle prq| = 106° 16'$.

Calculate $|\angle rpq|$ as accurately as the Tables allow.

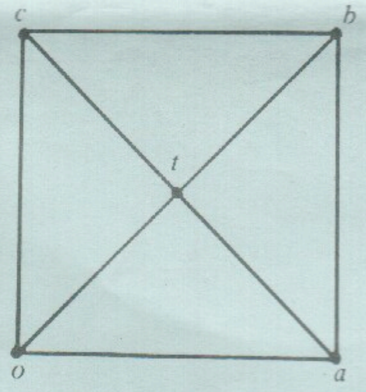


8. (a) $oabc$ is a square. o is the origin.

Using only the letters in the diagram name a vector equal to

(i) $\vec{oc} - \frac{1}{2}\vec{ac}$

(ii) $\vec{ot} - \vec{cb}$.



(b) $\vec{h} = 6\vec{i} - 8\vec{j}$ and $\vec{k} = 4\vec{i} - 3\vec{j}$.

(i) If $ohkm$ is a parallelogram, o being the origin, express $\vec{m}$ in terms of $\vec{i}$ and $\vec{j}$,

(ii) if $\vec{p} = \vec{k} + \alpha\,\vec{km}$, $\alpha \in \mathbf{R}$ and p is a point on the $\vec{j}$-axis, calculate the value of α. Express $\vec{p}$ in terms of $\vec{i}$ and $\vec{j}$ and calculate $|\vec{pm}|$.