LEAVING CERTIFICATE EXAMINATION, 1983

MATHEMATICS - ORDINARY LEVEL - PAPER II (300 marks)

MONDAY, 13 JUNE - MORNING, 9.30 to 12.00

Attempt **QUESTION 1** (100 marks) and **FOUR** other questions (50 marks each)

Marks may be lost if all your work is not clearly shown

1. (i) Express $\frac{2}{3}$ of 0·96 as a percentage of 5·12.

(ii) IR£10 is invested at 10% per annum compound interest. Calculate the amount after 2 years.

(iii) Express

$$\frac{11{\cdot}7 \times 0{\cdot}3}{1{\cdot}17 \div 0{\cdot}3}$$

in the form $a\ 10^n$, where $1 \leqslant a < 10$ and $n \in \mathbf{Z}$.

(iv) $x + 2$ is a factor of $x^2 - kx^2 + 4x - 8$. Find the value of k.

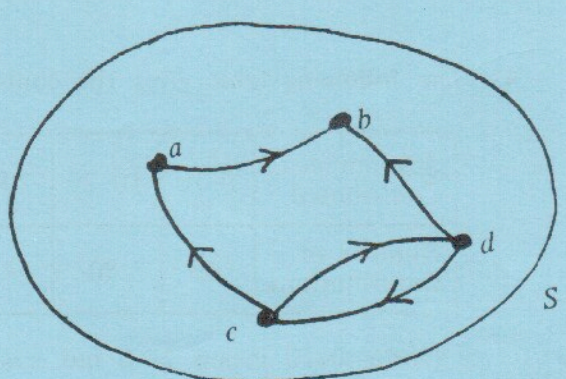
(v) x, $7\frac{1}{3}$, 11 are three numbers in geometric sequence. Find the value of x.

(vi) A person measures the length of a bar and gives 12·4 cm as its length. If the true length is within $\pm$ 10% of this answer, calculate the difference between his estimation of its possible maximum and possible minimum length.

(vii) By drawing a rough graph, or otherwise, find the range of values of $x \in \mathbf{R}$ for which

$$1 - 4x - 5x^2 \geqslant 0.$$

(vii) The diagram shows the couples of a relation S.
Write down the couples of the relation $S \circ S$.



(ix) Write out all the terms of the expansion of

$$(1 + px)^4.$$

Verify your answer by putting $x = \frac{1}{p}$.

(x) Find the coordinates of the local minimum of the curve

$$y = x^3 - 3x^2.$$

2. Plot on the Argand diagram the four complex numbers

$z_1 = 3 + 3i$, $z_2 = 2 - 2i$, $z_3 = z_2 - z_1$, $z_4 = z_2 + z_1$.

Evaluate $|z_3|$ and $|z_4|$ and investigate if the image of z_3 under the central symmetry in z_2 is z_4.

If $z_2 - tz_1 = ki$, where $t, k \in \mathbf{R}$, find t and k.

Use your diagram to verify that the line joining z_4 and z_2 cuts the imaginary axis at ki.

3. (a) Solve

$$\begin{aligned} 5x - 4y + z &= 3 \\ 3x + y - 2z &= 31 \\ x + 4y &= 21. \end{aligned}$$

(b) Verify that

$$8\binom{10}{2} = 3\binom{10}{3}.$$

How many terms of the binomial expansion of

$$(1{\cdot}01)^{10}$$

are necessary to show that

$$(1{\cdot}01)^{10} > 1{\cdot}104 \quad ?$$

4. The following table gives the contributions in IR£ of 1000 people to a fund:

Amount Contributed	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
Number of Contributions	60	80	110	160	200	180	140	70

(Note 0–10 means $\geqslant 0$ but less than 10 etc.)

Using these figures fill in the following cumulative frequency table:

Amount Contributed	< 10	< 20	< 30	< 40	< 50	< 60	< 70	< 80
Number of Contributions								

Draw a cumulative frequency curve to illustrate these data and use the curve to estimate

(i) the median contribution

(ii) the number of contributions of IR£65 or more

(iii) the percentage of contributions which were greater than or equal to IR£26 but less than IR£55.

5. The function $f: x \to x^3 - 2x^2 - 4x + 3$ is defined for $-2 \leqslant x \leqslant 4$, $x \in \mathbf{R}$.

Draw the graph of the function and use it to find, as accurately as possible,

(i) the value of $x > 0$ for which

$$f(x) = 3$$

and then use this value to estimate $\sqrt{5}$

(ii) the range of values of x for which the tangents to the curve have negative slope (gradient).

6. (a) In an arithmetic series the eighteenth term is 70 and the fourteenth term is equal to three times the fifth term.

Calculate the sum of the first 50 terms.

(b) Explain the meaning of each of the letters in the compound interest formula

$$A = P\left(1 + \frac{r}{100}\right)^n$$

and show that IR£22 is the amount of IR£$\frac{22}{(1{\cdot}02)^4}$ after 4 months at 2% per month compound interest.

Find to the nearest IR£ the least sum of money which must be lodged in a bank now at 2% per month compound interest to enable a withdrawal of IR£22 to be made at the end of each of the next 4 months from now.

(Take $(1{\cdot}02)^4 = 1{\cdot}08$, $(1{\cdot}02)^3 = 1{\cdot}06$, $(1{\cdot}02)^2 = 1{\cdot}04$)

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7. A manufacturer makes two models, A and B, of a piece of merchandise. His customers require at least 25 per month of A and 50 per month of B. He has only one finishing machine which works for 2 hours on each model A and for 1 hour on each model B and which can work for at most 340 hours per month. The maximum number of pieces he can manufacture in a month is 300.

If he makes a profit of IR£15 on each model A and IR£10 on each model B, how many of each should he make in a month to maximise his profit, assuming he can sell all he makes ?

8. (a) Differentiate from first principles

$$3x^2 - 1$$

with respect to x.

(b) (i) Find the coefficient of x^3 in the derivative of

$$(3x^2 - x - 1)(1 - 2x^2)$$

with respect to x.

(ii) Find the value of $\frac{dy}{dx}$ at $x = 1$ when

$$y = (3x^2 - x - 1)^{20}.$$

(c) Explain the geometrical meaning of $\frac{dy}{dx}$ and draw a rough graph of a function $y = f(x)$ which satisfies the conditions

$$y = 2 \quad \text{at } x = 0$$

$$\frac{dy}{dx} = 0 \quad \text{at } x = 0$$

$$\frac{dy}{dx} > 0 \quad \text{at } 0 < x \leqslant 2.$$