LEAVING CERTIFICATE EXAMINATION, 1983

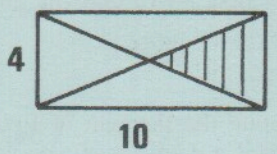
MATHEMATICS—ORDINARY LEVEL—PAPER I (300 marks)

FRIDAY, 10 JUNE—MORNING, 9.30–12.00

Attempt **Question 1** (100 marks) and **four** other questions (50 marks each)

Marks may be lost if all you work is not clearly shown.

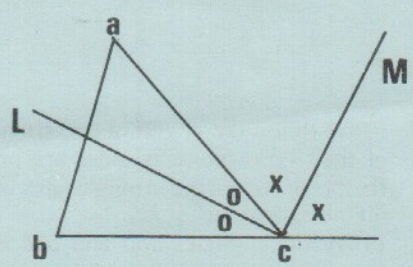
1. (i) A rectangular sheet of paper has a triangular piece cut from it (see diagram). Calculate the area of the shaded portion.



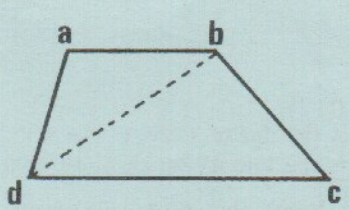
(ii) If $z = \frac{x}{2}(2y + x)$, express y in terms of z and x.

(iii) Using ratio, or otherwise, calculate the price per gallon, if 60 litres of petrol cost IR£33. (Take 1 gallon = 4·5 litres).

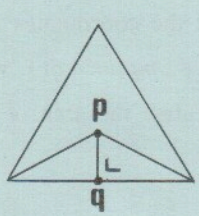
(iv) In the traingle abc, the angles at c are bisected, as in diagram, by M and L. Prove $M \perp L$.



(v) In the quadrilateral $abcd$, $ab \parallel cd$ and $|ab| = \frac{1}{2}|cd|$. If the area of $\triangle abd$ is k units, prove the area of bdc is $2k$ units.



(vi) The angles at the base of an equilateral triangle of side 2 are bisected and the bisectors meet at p (see diagram). Calculate $|pq|$.



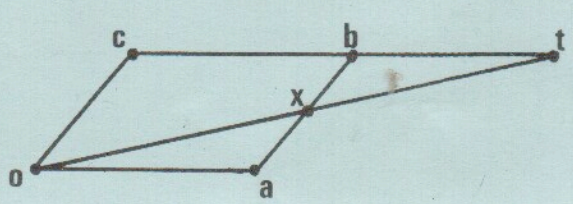
(vii) Find the equation of the line containing the point $(-1, 0)$ and parallel to

$$x + y = 0.$$

(viii) The points $(-1, 2)$, $(7, 2)$ are the end points of a diameter of a semi-circle. Write down the equation of the axis of symmetry of the semicircle.

(ix) Use Tables to find cos 1100°.

(x) $oabc$ is a parallelogram. x is the mid-point of $|ab|$. ox intersects cb in t.
Express $\vec{t}$ in terms of $\vec{a}$ and $\vec{c}$ where o is the origin.



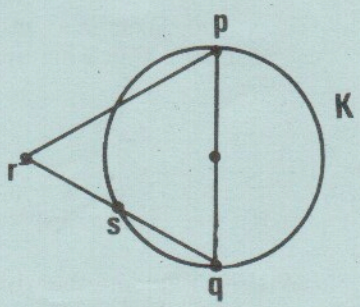
2. Liquid is taken from a cylindrical container by a ladle in the shape of a hemispherical bowl attached to a handle. The internal diameter of the container is 40 cm and that of the bowl is 10 cm. Calculate

   (i) the volume of liquid in the bowl, when full, in terms of π.

   (ii) the drop in the height of the liquid in the container after 33 full servings.

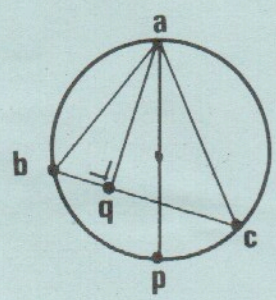
   (iii) the number of full servings remaining in the container if the depth of liquid unused is 5 cm.

3. (i) Prove that the measure of the angle at the centre of a circle is twice the measure of an angle at the circle standing on the same arc.
   Hence, prove that the angle in a semi-circle is a right angle.



   (ii) State the theorem of Pythagoras.
   pq is a diameter of a circle K. r is a point such that $|pr| = |pq|$.
   Prove that K bisects $[rq]$ at s.
   If t is a point between p and s prove that $|pq|^2 > |pt|^2 + |tq|^2$.

4. Prove that if the angles of two triangles are equal in measure, then the lengths of their corresponding sides are proportional.
   In the diagram the triangle abc is inscribed in a circle which has $[ap]$ as diameter and $aq \perp bc$.
   Prove the triangles abp and aqc are equiangular.
   If $|ab| = 4$, $|ac| = 5$ and $|ap| = 6$ calculate $|aq|$.



5. L is the line $x + 2y + 1 = 0$
   K is the line $4x - 2y + 19 = 0$.

   (i) Verify that $p\ (4, -2\frac{1}{2})$ is on L.

   (ii) Prove $L \perp K$.

   (iii) Find the coordinates of q, the intersection of L, K.

   (iv) Prove $|pq| = |qr|$, where r is the intersection of K and the y-axis.

   (v) Calculate the area of the triangle pqr.

6. (*a*) S_1 is the circle $x^2 + y^2 = \frac{10}{4}$. Find

(i) the length of the radius of S_1

(ii) the coordinates of the points at which S_1 intersects the x-axis

(iii) the equation of the tangent to S_1 at $(\frac{3}{2}, -\frac{1}{2})$.

(*b*) (0, 8), (−8, 2) are the end-points of a diameter of a circle S_2. Find

(i) the equation of S_2

(ii) the equation of the image of S_2 under a rotation, centre the origin, of +90°.

7. (*a*) Sketch the graph of the function

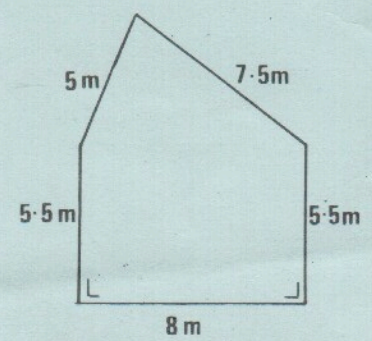
$$f : x \to 2 \sin x$$

in the domain $0 \leqslant x \leqslant 2\pi$.
Use your graph to write down

(i) the range of values of $f(x)$

(ii) the range of values of x for which $f(x)$ is decreasing.

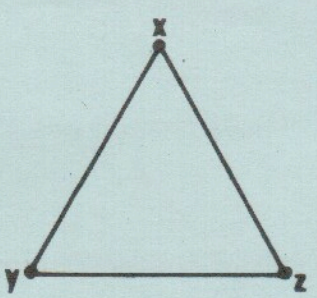
(*b*) The diagram represents the gable-end of a house. Calculate its area, as accurately as the tables allow using the measurements in the diagram.



8. (*a*) xyz is an equilateral triangle.

(i) Construct $\overrightarrow{xy} + \overrightarrow{zx}$ and indicate a couple which represents this vector.

(ii) Construct the point t such that $\overrightarrow{xy} + \frac{1}{2}\overrightarrow{yz} = \overrightarrow{zt}$.



(*b*) (i) $\vec{r} = 2\vec{i} - 5\vec{j}$ and $\vec{s} = -6\vec{i} + \vec{j}$ express the vectors $p = \vec{r} + \vec{s}$ and $\vec{q} = \vec{r} - \vec{s}$ in terms of $\vec{i}$ and $\vec{j}$. Illustrate $\vec{r}$, $\vec{s}$, $\vec{p}$ and $\vec{q}$ on a diagram.

(ii) Verify that $|\overrightarrow{op}| + |\overrightarrow{oq}| > |\overrightarrow{pq}|$.