MATHEMATICS – ORDINARY LEVEL – PAPER II (300 marks)

TUESDAY, 15 JUNE – MORNING, 9.30 to 12.00

Attempt **QUESTION 1** and **FOUR** other questions

Marks may be lost if all your work is not clearly shown

1. (i) An engine fuel consists of 8 litres of petrol mixed with $\frac{1}{2}$ litre of oil. Calculate the percentage of oil in the mixture correct to one place of decimals.

(ii) Find the value of $\qquad 2\pi\sqrt{\dfrac{k}{h}}$

when $k = 3\cdot 25$ and $h = 13\pi^2$.

(iii) The first three terms of an arithmetic sequence are

$$5x - 1, \quad 1, \quad x$$

Find the value of x.

(iv) Solve the simultaneous equations

$$\frac{1}{x} = 2$$

$$2x + \frac{3}{y} = 10.$$

(v) If $2^x \cdot 4^x = 16$, find x without the use of the Tables.

(vi) The following table shows the marks awarded in four subjects and the weights given to each subject:

	SUBJECTS			
Mark	2	3	5	6
Weight	3	4	1	2

Calculate the weighted average.

(vii) $f : \mathbf{R} \rightarrow \mathbf{R}$ is the function defined by

$$x \rightarrow \frac{x}{2} + 1 .$$

Write the function f^2 (i.e. $f \circ f$) in the form

$$x \rightarrow ax + b.$$

(viii) Write down three factors of: $\quad x^4 - y^4$.

(ix) $(p + q)^9$ is expanded in descending powers of p.   Evaluate the 4th term when $p = \frac{1}{2}$ and $q = 2$.

(x) Find the coordinates of the point on the curve $y = x^2 - 3x$ where the tangent to the curve is parallel to the line $y = x$.

(100 marks)

2. (a) If $-3 + 2i$ is a root of the equation
$$x^2 + ax + b = 0$$
where $a, b \in \mathbf{R}$ and $i = \sqrt{-1}$,
find the value of a and the value of b.

(b) Let $z_1 = 1 + 3i$ and $z_2 = 1 + i$.

Let $z_3 = z_2 + iz_1$ and $z_4 = \dfrac{z_1}{z_2}$.

Express z_3 and z_4 in the form $x + iy$.

Plot z_3 and z_4 on the Argand diagram and investigate if the image of z_3 under the central symmetry in the origin is the same as the image of z_4 under the axial symmetry in the real axis (i.e. x-axis).

(40 marks)

3. Complete the following sentence:

In a histogram the areas of the rectangles are proportional to the ....

Draw a histogram to represent the following frequency distribution:

Class	0 – 2	2 – 6	6 – 12	12 – 20
Frequency	16	16	24	44

(Note: 0 – 2 means $\geqslant 0$ but less than 2, etc.)

Assuming the data can be taken at the mid-interval values, calculate the mean and the standard deviation.

(40 marks)

4. The function
$$f : x \rightarrow x^3 - 6x + 2$$
is defined on the domain
$$-3 \leqslant x \leqslant 3, \quad x \in \mathbf{R}.$$
Draw the graph of f.

Use your graph to estimate the values of x for which
$$f(x) = 2$$
and hence estimate $\sqrt{6}$, giving a reason for your answer.

Investigate if $f(-\sqrt{6}) = -f(\sqrt{6})$
and show that
$$f(\sqrt{6}) + f(-\sqrt{6}) = 2f(0).$$

(50 marks)

5. (a) Prove that the sum of the first n terms of an arithmetic series is

$$\frac{n}{2}\left(2a + (n-1)\,d\right)$$

where a is the first term and d the common difference.

Find the sum of the first 60 terms of the arithmetic series

$$59 + 57 + 55 + \ldots\ldots$$

(b) Angela invested IR£3000 at 9·5% interest per annum. She was not charged tax on the first IR£150 interest but paid tax at the rate of 50% on the remainder. Calculate her net income (i.e. her income after tax was deducted).

John invested IR£3000 at 13·5% interest per annum and paid tax at the rate of 50% on all his interest. Calculate his net income.

If each was to invest the same sum of money in order to have equal net incomes, find this sum, assuming Angela pays tax.

(50 marks)

6. (a) Find the values of x for which

$$2x^3 - 3x^2 - 12x + 20 = 0.$$

(b) Verify that

$$\binom{5}{3} = \binom{5}{2}$$

and find $x \neq 6$ for which

$$\binom{20}{x} = \binom{20}{6}.$$

Write out all the terms in the expansion of

$$(1 + 3x)^5$$

and use this expansion to verify that

$$(-2)^5 = -32.$$

(50 marks)

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7. (i) Using the same axes and the same scales and taking $x, y \in \mathbf{R}$ graph each of the following half-planes:

$$A = \{(x, y) \mid x \geqslant 5\}$$

$$B = \{(x, y) \mid 3x + 4y \geqslant 120\}$$

$$C = \{(x, y) \mid 2x + y \leqslant 60\}$$

(ii) Indicate the set of points $D = A \cap B \cap C$.

(iii) Calculate the coordinates of the vertices of D (it is not sufficient to read these coordinates from your graph).

(iv) Find the couple $(x, y) \in D$ for which $100x + 40y$ is a maximum.

(50 marks)

8. (a) Differentiate from first principles $10 + x - x^2$ with respect to x.

(b) (i) Express $x^3 + \frac{2}{x}$ in the form $\frac{u}{v}$ and hence find the value of its derivative at $x = 1$.

(ii) If $y = (x^3 - 2x)^{12}$, find the value of $\frac{dy}{dx}$ at $x = -1$.

(c) Find where the curve

$$y = (x - 1)(x - 2), \ x \in \mathbf{R},$$

cuts the x-axis and draw a rough sketch of the curve.

Tangents are drawn to this curve at the points where it cuts the x-axis. Investigate if these two tangents are at right-angles to each other.

(50 marks)