

FRIDAY, 11 JUNE - MORNING, 9.30 - 12.00

Attempt QUESTION 1 and FOUR other questions

1982

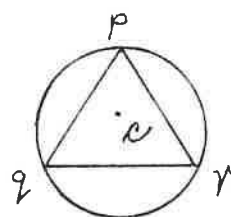
Marks may be lost if all your work is not clearly shown.

1. (i) The volume of a sphere is $\frac{32}{3} \pi \text{ cm}^3$. Calculate the length of the radius.

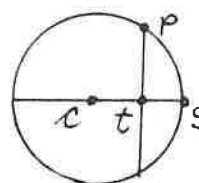
(ii) If $\frac{x-y}{y} = \frac{r}{s}$, express y in terms of x , r and s .

(iii) If 3 litres : 300 $\text{cm}^3 = 10 : x$, find x .

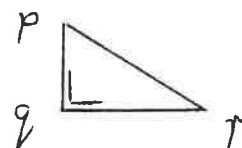
(iv) pqr is an equilateral triangle in a circle, centre c .
Prove $|\angle cpq| = 30^\circ$.



(v) c is the centre of a circle of radius 10.
Calculate $|pt|$ when $|ct| = |ts|$ and $|\angle ptc| = 90^\circ$.



(vi) pqr is a triangle, $|pq| = 5$, $|qr| = 12$,
 $|\angle pqr| = 90^\circ$. Calculate the length of
the radius of its circumcircle.



(vii) Calculate the area of the triangle oab if o is the origin, $a (-3, 2)$ and
 b is the image of a under the axial symmetry in the x -axis.

(viii) Find, as accurately as the Tables allow, the measure of the angle which the line
 $3x - 2y - 7 = 0$ makes with the positive sense of the x -axis.

(ix) Sketch the graph of $\cos x$ in the domain $0 \leq x \leq 2\pi$. State the range of
values of x for which $\cos x$ is negative.

(x) If $\vec{a} = 3\vec{i} + 3\vec{j}$ and $\vec{b} = 18\vec{i} - 8\vec{j}$, write \vec{ab} in terms of \vec{i} and \vec{j} .

(100 marks)

2. The base of a right circular cone has a radius of length 5 cm. The height of the
cone is 12 cm. Calculate the volume in terms of π .

The inverted cone is filled with water. The water then drips from the vertex at
the rate of $0.628 \text{ cm}^3/\text{sec}$. Taking $\pi = 3.14$, calculate the time in seconds until the
cone is empty, assuming the volume of water to be the same as the volume of the cone.

If all the water dripped into a dry cylindrical can of diameter 10 cm in length,
calculate the height of water in the can.

(40 marks)

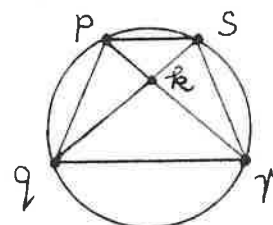
3. (a) Prove that triangles on the same base and between the same parallels are equal
in area.

(b) p, q, r, s are points of a circle and $ps \parallel qr$.
From the diagram, name

(i) three pairs of triangles, each pair being
equal in area

(ii) a ratio of line segments which is equal to
 $|pk| : |kr|$

(iii) a ratio of line segments which is equal to
 $\frac{\text{area of the triangle } pqk}{\text{area of the triangle } pks}$



Prove that pks is an isosceles triangle.

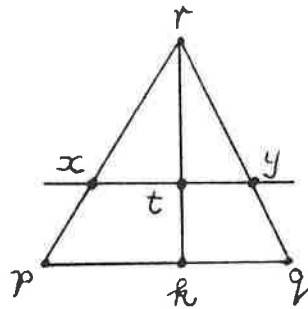
(40 marks)

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4. (i) Prove that two sides of a triangle are divided proportionally by a line drawn parallel to the third side.
- (ii) Given a line segment $[pq]$ such that $|pq| = 7$ cm, explain how a point $k \in [pq]$ is constructed such that $|pk| : |kq| = 5 : 4$. Indicate your construction by a diagram.

1997

- (iii) If $xy \parallel pq$ and k divides $[pq]$ in the ratio $5 : 4$, prove that t also divides $[xy]$ in the same ratio.



(50 marks)

5. L is the line $2x - 3y - 7 = 0$.
Verify that the point $a(2, -1)$ is in L .
Find (i) the slope of L ,
(ii) b , the point of intersection of L and the y -axis,
(iii) the equation of the image of L under the translation
 $a(2, -1) \rightarrow d(0, 2)$.

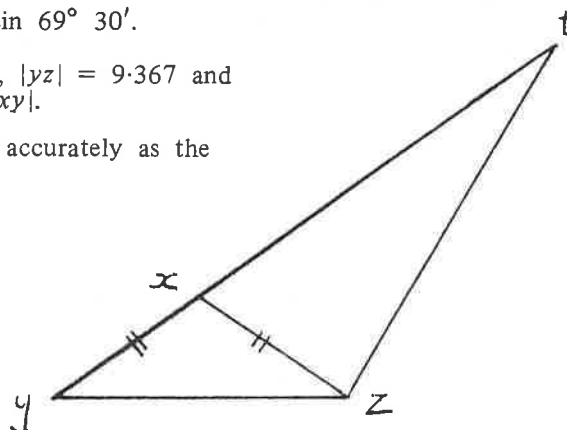
Prove that the line joining a and d is perpendicular to L .
Name the coordinates of c in the rectangle $abcd$.

(50 marks)

6. (a) K is a circle, centre $(0, 0)$. The length of the radius is $\sqrt{5}$. Write the equation of K .
Prove that the line $L : y = 2x + 5$ is a tangent to K and find the coordinates of $K \cap L$.
Find the equation of the tangent to K at the point $(2, -1)$.
- (b) The vertices of a square $abcd$ are $a(1, 2)$, $b(-3, -6)$, $c(9, -12)$ and $d(13, -4)$. Find the equation of the circle drawn through the four vertices of the square. What is the equation of the image of this circle under the axial symmetry in the y -axis?

(50 marks)

7. (i) Using $\sin(A - B) = \sin A \cos B - \cos A \sin B$, verify giving reasons, that $\sin(180^\circ - 69^\circ 30') = \sin 69^\circ 30'$.
- (ii) In the triangle xyz , $|xy| = |xz|$, $|yz| = 9.367$ and $|\angle xyz| = 34^\circ 45'$. Calculate $|xy|$.
If $|ty| = 25.7$, calculate $|tz|$ as accurately as the tables allow.



(50 marks)

8. (a) If $\vec{o} = 0\vec{i} + 0\vec{j}$, $\vec{p} = 3\vec{i} - 4\vec{j}$ and $\vec{q} = -2\vec{i} - 2\vec{j}$, express in the form $a\vec{i} + b\vec{j}$ where $a, b \in \mathbb{R}$

(i) $\vec{p} + \vec{q}$ (ii) $\vec{p} + \frac{1}{2}\vec{q}$ (iii) $\vec{p} - \vec{q}$ (iv) $\frac{\vec{p}}{|\vec{p}|}$

- (b) $oabc$ is a parallelogram. Its diagonals intersect at t .
 s is the mid-point of $[ab]$.

Write (i) \vec{s} (ii) \vec{t} and (iii) \vec{g} in terms of \vec{a} and \vec{c} .

