

AN ROINN OIDEACHAIS

M.48

LEAVING CERTIFICATE EXAMINATION, 1981

MATHEMATICS - ORDINARY LEVEL - PAPER II (300 marks)

MONDAY, 15 JUNE - MORNING, 9.30 to 12.00

Attempt QUESTION 1 and FOUR other questions

Marks may be lost if all your work is not clearly shown.

1. (i) IR£2.85 pocket money is divided between two sisters in the ratio 3 : 2. How much does each get ?
- (ii) Find the compound interest on IR£1000 after two years at 10% per annum.
- (iii) If $b = -5$, find the value of $\frac{-b + \sqrt{b^2 + 144}}{8}$
- (iv) Find the value of the fifth term in the expansion of $(1 + x)^{20}$ in ascending powers of x when $x = \frac{1}{10}$.
- (v) Solve the simultaneous equations
 $x = 3;$ $3x + 3y = 5;$ $x + y + z = 3\frac{1}{2}.$
- (vi) The n th term of a sequence is given by $\frac{(-1)^n}{2n - 3}$. Write out the first three terms and find their sum.
- (vii) For what range of values of $x \in \mathbf{R}$ is $3x^2 + x - 2 \leq 0$?
- (viii) k is the mean of the four numbers a, b, c, d . Find in terms of k the mean of the four numbers $a + 1, b + 1, c + 1, d + 1$.
- (ix) $f : \mathbf{R} \rightarrow \mathbf{R}$ is the function defined by $x \rightarrow 3x + 2$. Find the value of $f^{-1}(-2)$.
- (x) T is a tangent to the graph of $y = 2x^2 - 3x + 1$ at the point $(1, 0)$. Find the measure of the angle that T makes with the positive sense of the x -axis.

(100 marks)

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2. (a) On the Argand diagram plot the point p which represents the complex number $1 + 2i$. What complex number in each of the following is represented by the image of p under
- the central symmetry in the origin
 - the axial symmetry in the real axis (i.e. the x -axis) after the central symmetry in the origin
 - the axial symmetry in the imaginary axis (i.e. the y -axis) ?

(b) Find the values of $x, y \in \mathbb{R}$ such that

$$(x - iy) + (y + ix) = 1 - 5i \quad \text{where } i = \sqrt{-1}.$$

Using these values of x and y express

$$\frac{x - iy}{y + ix}$$

in the form $a + ib$, where $a, b \in \mathbb{R}$, and hence, or otherwise, find the value of

$$\left| \frac{x - iy}{y + ix} \right|.$$

(40 marks)

3. If $f(x) = 4x^3 - 13x + 6$, complete the following table

x	-2	-1	0	1	2
$f(x)$		15			12

and hence draw a graph of the function

$$f : x \rightarrow 4x^3 - 13x + 6$$

in the domain $-2 \leq x \leq 2$, $x \in \mathbb{R}$.

Find from your graph as accurately as you can the values of x for which

- $f(x) = 0$
- $f(x) = 6$

and the range of values of x for which $f(x) \leq 0$.

(40 marks)

4. By fitting an electronic device to its shopping baskets a supermarket records the time each customer spends shopping on a day in which there are 660 customers. The data is summarised in the following cumulative frequency distribution:

Time in Minutes	less than 10	less than 20	less than 30	less than 40	less than 50	less than 60	less than 70	less than 80
Number of Customers	40	90	160	290	450	560	630	660

Draw the cumulative frequency graph, plotting the times along the horizontal axis. Estimate from your graph the number of customers who spent

- less than $\frac{1}{4}$ hour in the supermarket
- more than $\frac{3}{4}$ hour in the supermarket.

Also indicate the median shopping time for that day.

Use the given data to complete the following grouped frequency distribution table:

Time in Minutes	0 - 20	20 - 40	40 - 60	60 - 80
Number of Customers	90		270	

(Note: 0 - 20 means > 0 but less than 20 etc.)

Taking the times at the mean interval values evaluate, to the nearest minute, the mean shopping time for that day.

5. (a) Using the usual notation write down the formulae for T_n , the n th term and for S_n , the sum of the first n terms, of an arithmetic series. 1981

In an arithmetic series $T_6 = 208$ and $T_{11} = 183$.

Calculate S_{40} .

(b) IR£1 is invested at 10% per annum, compound interest. Write down the amount that this investment comes to after 1 year and show that it amounts to IR£6.73 after 20 years.
 IR£1 is invested at 10% per annum compound interest at the beginning of each year until IR£20 in all has been invested. Calculate the amount that this investment comes to one year after the final IR£1 has been invested.

(50 marks)

6. (a) Solve the simultaneous equations

$$x + xy = 9$$

$$\frac{1}{x} + \frac{y}{x} = 25$$

(b) Evaluate $\binom{8}{3}$ and write down a problem for which $\binom{8}{3}$ is the solution.

Verify that $\binom{8}{2} + \binom{8}{3} = \binom{9}{3}$

(c) Write out the first three terms of the expansion of

$$(1 - 2x)^8$$

in ascending powers of x .

Use your result to evaluate $(0.998)^8$, correct to two places of decimals.

(50 marks)

7. A manufacturer has a ready market for components of two types - type A and type B.

The maximum machine time available for their manufacture is 9 hours.

The maximum energy available is 180 units.

The following table shows the time and energy requirement for each type of component.

Component	Type A	Type B
Minutes per component	12	10
Energy units per component	3	4

If the profit on each type A component is IR£7 and on each type B component is IR£10, calculate the number of components of each type that should be made in order to have a maximum profit.

(50 marks)

8. (a) Differentiate x^3 with respect to x from first principles.

(b) Find the coordinates of the local maximum of the function

$$y = x^3 - 9x^2 + 24x - 20, \quad x \in \mathbb{R}$$

(c) Find the value of $\frac{dy}{dx}$ at $x = 0$ in the case of each of the following:

(i) $y = \frac{x^2 + x - 2}{x^2 - x - 2}$

(ii) $y = (1 - x^2)^{13}$

(50 marks)