LEAVING CERTIFICATE EXAMINATION, 1981

LC

MATHEMATICS - ORDINARY LEVEL - PAPER I (300 marks) (1981)

WEDNESDAY, 10 JUNE - MORNING, 9.45 - 12.15

Attempt **QUESTION 1** and **FOUR** other questions

Marks may be lost if all your work is not clearly shown.

1. (i) A wire two metres in length is bent into the shape of a rectangle. The shorter side is 26 cm in length. Calculate the area enclosed by the rectangle.

(ii) If $A = 2\pi r\,(h + r)$, $r \neq 0$, express h in terms of A, π, r.

(iii) A prize fund of IR£3900 is divided as follows:

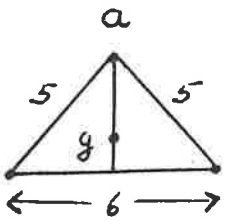
the first prize is half the fund,
the second prize is two-thirds the first prize,
the third prize is what remains.

How much is the third prize worth ?

(iv) Given a line segment 1 unit in length, show how to draw line segments $\sqrt{2}$ and $\sqrt{3}$ units in length.

(v) A circle has a diameter of length 10 cm. Express in radians the size of an angle subtended at the centre by an arc of length 6·5 cm.

(vi) In the triangle, g is the point of intersection of the medians (centroid).
Calculate $|ag|$.



(vii) The slope of the line through the points $(2t, 6)$ and $(3t, 4)$ is 4. Calculate t.

(viii) Find the coordinates of the point of intersection of the two lines

$$3x + 4y = 8$$
$$2x + 3y = 7.$$

(ix) If $\sin A = \frac{4}{5}$ and $\cos B = \frac{5}{13}$, find in the form $\frac{a}{b}$ the value of $\sin(A + B)$. (See Tables P. 9)

(x) If $\vec{p} = 15\vec{i} + 23\vec{j}$, name a vector $\vec{q}$ in terms of $\vec{i}$ and $\vec{j}$ such that o, p and q are collinear, where o is the origin.

(100 marks)

2. Calculate the volume of a sphere of diameter 9 cm in length (take $\pi = 3{\cdot}14$). Two such spheres, placed on the bottom of a rectangular closed box, fit into the box exactly. The rest of the space in the box is filled with foam packing.
Calculate the volume of the space filled by the foam packing, correct to the nearest cm^3.

(40 marks)

3. (a) Prove that the measure of the angle at the centre of a circle is twice the measure of an angle at the circle standing on the same arc.

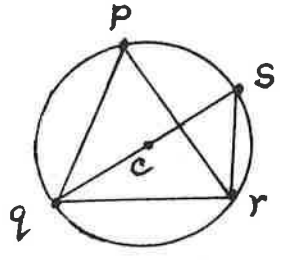
(b) p, q, r, s are points of a circle, centre c, see diagram.
Name an angle equal in measure to

(i) $2\,|<rps|$

(ii) $|<qsp|$

(iii) a right angle

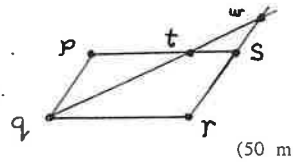
If $|pq| = |pr|$ and $|<qsr| = 36°$, calculate $|<prs|$.



4. The angles of two triangles are equal in measure. Prove that the lengths of the corresponding sides are proportional.

$p\,q\,r\,s$ is a parallelogram and t is any point in $[ps]$, as in diagram.
Prove $\Delta p\,q\,t$ and $\Delta r\,w\,q$ are equiangular.

If $|pt| = \frac{2}{3}|ps|$, prove $|rs| = \frac{2}{3}|rw|$.



(50 marks)

5. l, m, n are the three points $(-2, 3)$, $(1, 6)$ and $(5, 2)$ respectively.
The translation $\vec{lo}$ maps $m \to p$ and $n \to q$ where o is the origin. Calculate

(i) the coordinates of p and of q
(ii) $|m\,n|$
(iii) the area of the $\Delta\, lmn$
(iv) the equation of the line L through o perpendicular to pq
(v) the image of q under the composition of axial symmetries $S_L \circ S_X$ where X is the x-axis.

(50 marks)

6. (a) The point $(2, \sqrt{21})$ is on a circle K, centre $(0, 0)$. Find the equation of K.
The line $x - 2y + 5 = 0$ cuts K at p and q. Calculate $|pq|$.

(b) The image of the circle $(x - 2)^2 + (y + 6)^2 = 16$
under the central symmetry in (r,s) is $x^2 + y^2 - 6y = 7$.
Calculate the coordinates (r, s).

(50 marks)

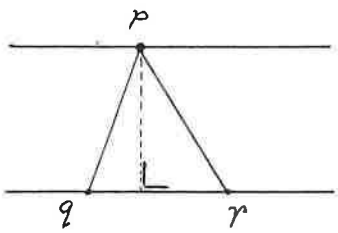
7. (a) Draw the graph of the periodic function

$$f : x \to \sin 2x$$

in the domain $0 \leqslant x \leqslant \pi$, $x \in \mathbb{R}$.

Without further calculation sketch the graph of f in $\pi \leqslant x \leqslant 2\pi$ and estimate the values of x for which $f(x) = 0{\cdot}5$ in $0 \leqslant x \leqslant 2\pi$.

(b) A surveyor sighted the point p, on one side of a ravine, from two points q and r on the other side (see diagram). If the edges of the ravine are straight and parallel and $|qr| = 71$ m, $|\angle pqr| = 79°\ 30'$ and $|\angle prq| = 64°\ 50'$, calculate to the nearest metre the width of the ravine at p.

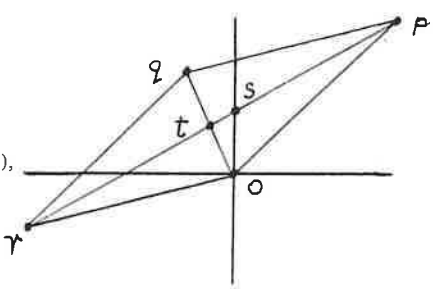


(50 marks)

8. (i) If $\vec{p}$ and $\vec{q}$ are vectors and o is the origin, show by a diagram that $\vec{pq} = \vec{q} - \vec{p}$.

(ii) In the diagram, $o\,p\,q\,r$ is a parallelogram,
$\vec{p} = 3\vec{i} + 3\vec{j}$, $\vec{q} = -\vec{i} + 2\vec{j}$.
Express in terms of $\vec{i}$ and $\vec{j}$
$\vec{pq}$, $\vec{qr}$, $(\vec{op} + \vec{pq} - \vec{qr})$.

If $\vec{ts} = m\,\vec{rp}$ and $\vec{s} = o\vec{i} + k\vec{j}$ (see diagram), find the value of m and the value of k where $m, k \in \mathbb{R}$.



(50 marks)