AN ROINN OIDEACHAIS M.47

LEAVING CERTIFICATE EXAMINATION, 1980

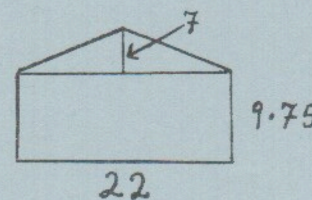
8904

MATHEMATICS - ORDINARY LEVEL - PAPER I (300 marks)

FRIDAY, 13 JUNE - MORNING, 9.30 - 12.00

Attempt **QUESTION 1** and **FOUR** other questions

1. (i) An envelope has measurements, in cm, as in the diagram. Calculate the surface area of the front of the envelope, as shown.



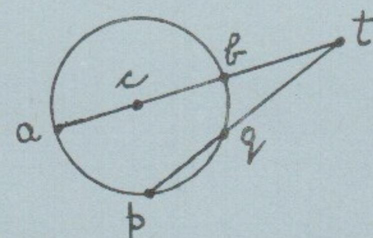

(ii) If $t = k(1 + \frac{v}{c})$, express $v$ in terms of $t$, $k$ and $c$, where $k \neq 0$ and $c \neq 0$.

(iii) The area of the triangle with vertices $(0, 0)$, $(5t, 3t)$, $(t, 2t)$ is 14. Find the two possible values of $t$.

(iv) Find the equation of the line which passes through the point $(-3, -2)$ and makes an angle measuring $16^\circ\ 42'$ with the positive sense of the $x$-axis.

(v) A glass rod falls and breaks into three pieces whose lengths are in the ratio of 7 : 5 : 3. If the sum of the lengths of the two smaller pieces is 16 cm, find the length of the third piece.

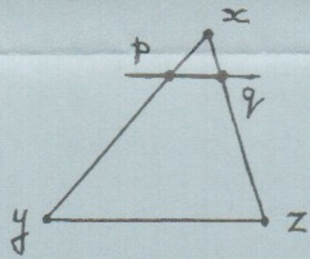
(vi) The centre of the circle is $c$ and $|cb| = |bt|$.
If $|pq| = 3$ and $|qt| = 9$, calculate the length of the radius of the circle.



(vii) In $\Delta xyz$, $pq \parallel yz$

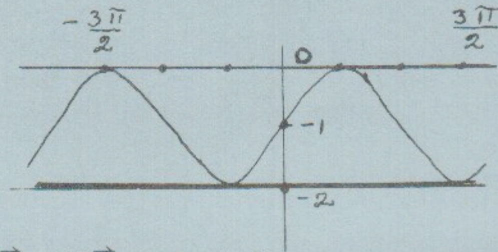
$|xy| = 14, \quad |xz| = 10, \quad |xq| = 2$

Calculate $|py|$.




(viii) If $\cos A = \frac{12}{13}$, $0 < A < \frac{\pi}{2}$ find the value of $\sin 2A$

(ix) The graph of a periodic function consists of repetitions of this diagram. What is the period and range of the function ?




(x) $\vec{o}$ and $\vec{t}$ are the vectors $0\vec{i} + 0\vec{j}$ and $5\vec{i} - 12\vec{j}$, respectively.
If $k \in [ot]$ such that $|ok| = 1$, express $\vec{k}$ in terms of $\vec{i}$ and $\vec{j}$. (100 marks)

2. The diagram, not to scale, shows a swimming pool of length 50 m and width 20 m. The pool has plane vertical walls and a plane sloping floor such that its depth at one end is 1 m and at the other end is 6 m.

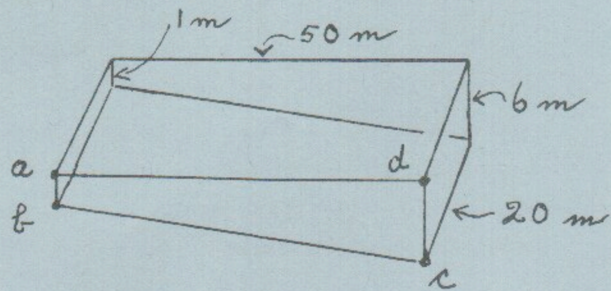
Calculate

(i) the average speed in metres per second of a swimmer who swims 5 lengths of the pool in 3 minutes 20 seconds,

(ii) the surface area of the wall $abcd$,

(iii) the volume of water in the pool,

(iv) the cost of heating the water for 9 hours if the average cost per cubic metre per hour is 0·034p.

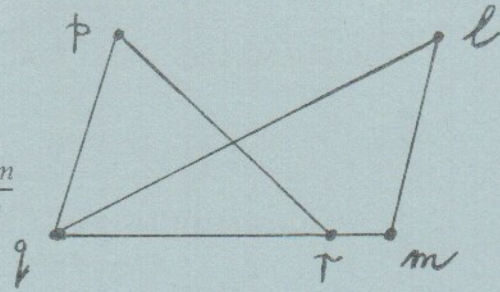



(40 marks)

3. Prove that the areas of two triangles of equal height are proportional to the lengths of their bases.

In the diagram, $qrp$ and $qml$ are two triangles.
$pl \parallel qm$, $lm \parallel pq$ and $|qr| : |rm| = 5 : 1$.
State the value of each of the following ratios:

(i) $\dfrac{\text{area } \Delta\, pqr}{\text{area } \Delta\, lqm}$ (ii) $\dfrac{\text{area } \Delta\, prl}{\text{area } \Delta\, pml}$ (iii) $\dfrac{\text{area } \Delta\, lqm}{\text{area } \Delta\, lrm}$




(40 marks)

4. (a) $[ab]$ and $[cd]$ are two chords of a circle intersecting in $k$ inside the circle, prove

$$|ak| \,.\, |kb| = |ck| \,.\, |kd| \,.$$

(b) From a point $p$ outside a circle, two tangents $pk$, $pt$ are drawn to touch the circle in $k$ and $t$, respectively.

Prove $|pk| = |pt|$ .

(50 marks)

5. $L$ is the line $5x + 2y + 3 = 0$ and $K$ is the line $2x - 5y + 7 = 0$.

(i) Verify that $(4, 3) \in K$.
(ii) Prove $L \perp K$.
(iii) Find the point of intersection of $L$ and $K$.
(iv) Calculate the image of $(4, 3)$ by $S_L$, the axial symmetry in $L$.
(v) Find the equation of the image of $L$ by $S_O$, the central symmetry in the origin.

(50 marks)

6. (i) Find the coordinates of the points of intersection of the line $x + y + 1 = 0$ and the circle $S : x^2 + y^2 = 25$.

(ii) $a(3, 4)$, $b(9, 12)$, $c(2, 11)$ are the vertices of the $\Delta abc$ which is right angled at $c$. Find the equation of the circle $K$ having $[ab]$ as diameter.

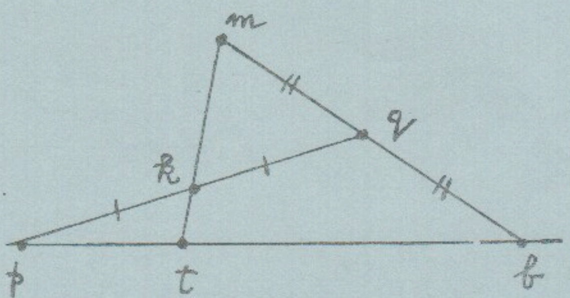
(iii) If the translation $(2, 11) \rightarrow (h, k)$ maps $K$ of part (ii) to $S$ of part (i) find $(h, k)$.

(50 marks)

7. A straight beam $[bm]$ is supported by two straight bars $[pq]$ and $[tm]$, 36 m and 34 m long respectively. The bars are joined at $k$, the midpoint of $[pq]$.

$|\angle mkq| = 34° \, 9'$, $|\angle qpb| = 30°$ calculate $|kt|$.

If $q$ is the midpoint of $[mb]$, calculate, correct to the nearest metre, $|bm|$.




(50 marks)

8. (a) If $\vec{a} = 5\vec{i} + 7\vec{j}$ and $\vec{b} = 3\vec{i} - \vec{j}$

(i) express $\vec{a} + \vec{b}$ in terms of $\vec{i}$ and $\vec{j}$

(ii) illustrate the vectors $\vec{a}$, $\vec{b}$, $\vec{a} + \vec{b}$ on a diagram

(iii) evaluate the modulus of $\vec{a} + \vec{b}$

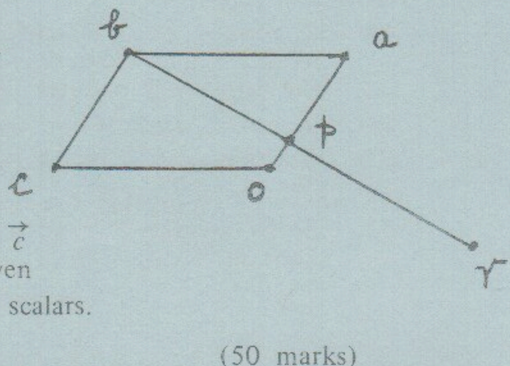
(iv) find $k$ and $t$ where $k(\vec{a} + \vec{b}) = 16\vec{i} + (t + 2)\vec{j}$

(b) In the diagram $oabc$ is a parallelogram. $p$ is a point on $oa$ such that $|op| : |pa| = 2 : 3$

$[bp]$ is produced to $r$ such that $|bp| = |pr|$.

Taking $\vec{o}$ as origin (i) express $\vec{bp}$ in terms of $\vec{a}$ and $\vec{c}$

(ii) find the values of $h$ and $k$ given $\vec{r} = h\vec{a} + k\vec{c}$, $h$ and $k$ being scalars.




(50 marks)