LEAVING CERTIFICATE EXAMINATION, 1979

MATHEMATICS - ORDINARY LEVEL - PAPER II   (300 marks)

WEDNESDAY, 13 JUNE - MORNING, 9.30 to 12.00

Attempt **QUESTION 1** and **FOUR** other questions

1. (i) Express in decimal form

$$\frac{3}{4} - \frac{5}{8} .$$

(ii) Three numbers in geometrical sequence are

$$\frac{17}{5}, \quad x, \quad \frac{5}{17} .$$

Find a value of x.

(iii) Solve for x:

$$\frac{3}{2x + 3} + \frac{5}{2x - 5} = \frac{4(2x + 1)}{(2x + 3)(2x - 5)}$$

(iv) If
$$f : x \rightarrow 2(x + 1)$$
$$g : x \rightarrow -3(x - 1),$$
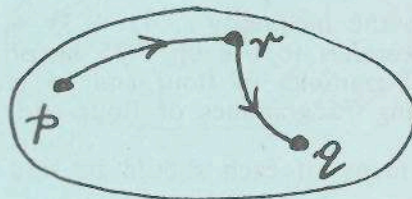find $gf(x)$, where gf is the composite function.

(v) Solve the simultaneous equations:

$$x - y + z = 4$$
$$x + y - z = 2$$
$$x + y + z = 8 .$$

(vi) Find the three values of x which satisfy

$$x^3 - 2x^2 - x + 2 = 0 .$$

(vii) Copy the diagram shown and complete it to show an order relation.

(viii) Express the sum of the first 40 terms of the geometric series

$$1 + \tfrac{1}{2} + \tfrac{1}{4} + \cdots\cdots\cdots$$

in the form $2 - (\tfrac{1}{2})^k$.

(ix) The number of games in which one goal or less, two goals or less, etc, were scored in 25 games is

	one goal or less	two goals or less	three goals or less	four goals or less
Number of games	10	15	20	25

Calculate the number of games in which at least 3 goals were scored.

(x) Find the coordinates of the local maximum of the curve $y = 2x^3 + 3x^2$.

(100 marks)

2. Represent the complex numbers $z_1 = 2 + i$ and $z_2 = 2 - i$ on an Argand diagram.
Calculate $\tfrac{1}{2}(z_1 + z_2)$.
Verify that
$$|z_1 \cdot z_2| = |z_1| \cdot |z_2|.$$

Express $\dfrac{z_1}{z_2}$ in the form $a + ib$ and find k such that $|z_1| = k \left| \dfrac{z_1}{z_2} \right|$.

(40 marks)

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3. The table shows the number of people who entered a stadium in the time intervals shown.

Time interval	6.00–6.30	6.30–7.00	7.00–7.30	7.30–8.00	8.00–8.30	8.30–9.00
Number of people (i.e. frequency)	200	300	500	1200	1800	2000

Copy and fill in the following cumulative frequency table which shows the number of people who had entered before the stated times.

Time	Before 6.30	Before 7.00	Before 7.30	Before 8.00	Before 8.30	Before 9.00
Number of people who had entered						

Draw a cumulative frequence curve and use the curve to estimate
(i) the number of people who had entered the stadium between 8.15 and 9.00
(ii) the time interval, beginning at 7.30, during which 4500 people entered the stadium
(iii) the time by which half the total audience had entered.

(40 marks)

4. The function $f : x \to x^3 - 4x^2 - 7$ is defined on the domain $-2 \leqslant x \leqslant 5$ for $x \in$ R. Draw a graph of f and use the graph to find as accurately as you can the range of values of x for which
(i) $f(x) < 0$ and increasing
(ii) $f(x) + 10 < 0$ when $x > 0$.

By drawing an appropriate straight line, find approximately the values of x for which
$$f(x) = 5(x - 4) .$$

(50 marks)

5. (a) In the arithmetic series $50 + 47 + 44 + \cdots$
find
(i) T_{100}, the one hundredth term
(ii) S_{100}, the sum of the first 100 terms.

(b) A sum of money borrowed for two years at 10% per annum compound interest is repayed together with the interest in equal instalments of £396 at the end of each of the two years. How much, to the nearest penny, was borrowed ?

(50 marks)

6. Graph the inequality $4x + 3y \leqslant 90$ for $x, y \in$ R.
A baker has to use up 2·25 kg of flour and 0·45 kg of raisins. He can make cakes each using 100 grammes of flour and 15 grammes of raisins. These sell at 96p each. Richer cakes, each using 75 grammes of flour and 30 grammes of raisins can also be made. These sell at £1.20 each.
How many of each should be sold to show the greatest profit, assuming that all cakes were sold ?
If cost amounted to £14.48, what was this profit ?

(50 marks)

7. Evaluate $\binom{12}{3}$ and $\binom{12}{9}$.

Write out the first four terms and the last four terms of the binomial expansion of $(1 + x)^{12}$ in increasing powers of x.

Evaluate $(0·998)^{12}$ correct to four places of decimals.

(50 marks)

8. (a) Differentiate $x^2 - x$ with respect to x from first principles.
For what value of x is the tangent to the graph of $y = x^2 - x$ parallel to x–axis ?

(b) Find $\frac{dy}{dx}$ for $y = x^3 - 3x + 7$. For what range of values of x is $\frac{dy}{dx} < 0$?

(c) A tangent is drawn to the graph of $y = \dfrac{x^2 - 1}{x^2 + 1}$

at the point $(1, 0)$. Find the size of the smaller angle that this tangent makes with the x-axis.

(d) Find the value of the derivative of $(7 + \frac{1}{3} x^3 - 5x^5)^4$ when $5x + 1 = 0$.

(50 marks)