AN ROINN OIDEACHAIS M.47

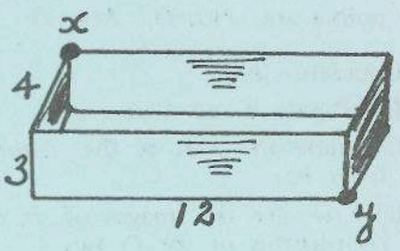
LEAVING CERTIFICATE EXAMINATION, 1979

MATHEMATICS–ORDINARY LEVEL–PAPER I (300 marks)

MONDAY, 11 JUNE–MORNING, 9.30 to 12.00

Attempt **QUESTION 1** and **FOUR** other questions

1. (i) A rectangular match-box has measurements 3, 4, 12 as in diagram. Calculate $|xy|$.



(ii) If $y = m(x - k)$, express k in terms of y, m, x where $m \neq 0$.

(iii) The translation $(-35, 45) \rightarrow (0, 0)$ maps $(4, -3) \rightarrow (p, q)$. Find the value of p and the value of q.

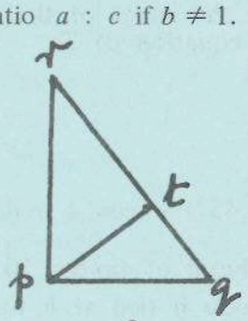
(iv) Find the slope of a line which is perpendicular to the line $3x + 4y - 7 = 0$.

(v) If $4a = 3(b - 1)$ and $2b = 3c + 2$, calculate the ratio $a : c$ if $b \neq 1$.

(vi) In the diagram

$|\angle rpq| = 90°$ and $pt \perp rq$.

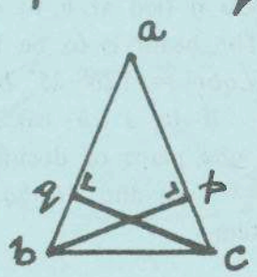
If $|qp| = 6$ and $|qt| = 4$, evaluate $|qr|$.



(vii) In the $\triangle abc$,

$bp \perp ac$, $cq \perp ab$, $|bp| = |cq|$.

Prove $|ab| = |ac|$.



(viii) Express $\frac{11\pi}{18}$ radians in degrees.

(ix) If $\cos A = -0{\cdot}5$, find the values of A for $0° \leqslant A \leqslant 360°$.

(x) $oabc$ is a parallelogram. If o is the origin, $\vec{a} = 2\vec{i} - 3\frac{1}{2}\vec{j}$, $\vec{b} = 7\frac{1}{4}\vec{i} - 3\vec{j}$, express $\vec{c}$ in terms of $\vec{i}$ and $\vec{j}$.

(100 marks)

2. The diameter of a solid hemisphere made of lead is 12 cm in length. Express its volume in terms of π.

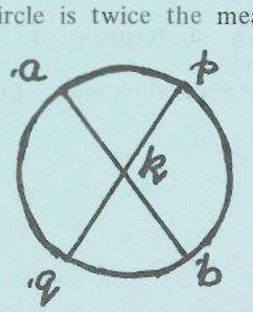
The lead is melted and a solid cone and a solid cylinder are formed from it. If the base of each is of radius 3 cm and the height of each is 6 cm, calculate the percentage of lead that remains unused.

(40 marks)

3. Prove that the measure of the angle at the centre of a circle is twice the measure of an angle at the circle standing on the same arc.

$[ab]$ and $[pq]$ are two diameters of a circle as in diagram.

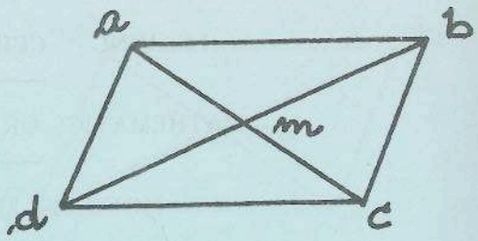
Prove that $|\angle qab| = |\angle qpb|$.
Hence, or otherwise, prove that $aq \parallel pb$.



(40 marks)

OVER →

4. Prove that the medians of a triangle are concurrent.



$abcd$ is a parallelogram.
p is the centroid of the $\Delta\, abd$.
q is the centroid of the $\Delta\, abc$.
Prove that $pq \parallel ab$ and find the ratio $|pq| : |ab|$.

(50 marks)

5. Three points are $a(0, 3)$, $b(4, 1)$, $c(-2, -1)$.

(i) Calculate $|bc|$.

(ii) Investigate if $ab \perp ac$.

(iii) Calculate the area of the Δabc and hence, or otherwise, find the distance of a from bc.

(iv) If v, w are the images of b, c, respectively, under the axial symmetry in the y-axis, find the coordinates of $bc \cap vw$.

(50 marks)

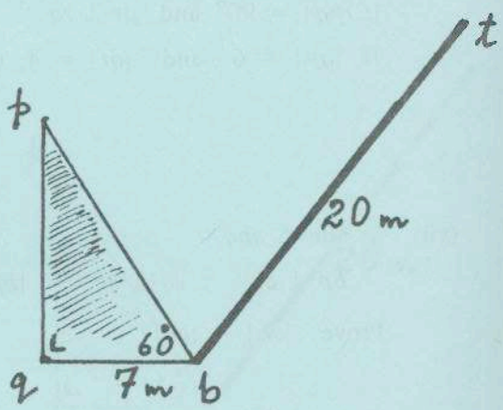
6. (a) Find where the line $x - 3y = 10$ cuts the circle $x^2 + y^2 = 10$ and investigate if this line is a tangent to the circle.

(b) (h, k) are the coordinates of the centre of the circle $x^2 + y^2 - 6y = 7$. Find (h, k) and the radius of the circle.
Let T be the image of the circle under the central symmetry in the point $(4, 0)$.
Find the equation of T.

(50 marks)

7. (a) Prove
$\sin(A + 45°) - \cos(A + 45°) = \sqrt{2}\ \sin A$

(b) $[bt]$ is a beam of timber 20 m in length which is tied at b to a triangular frame. The beam is to be kept in the position $|\angle qbt| = 126° 25'$ by an iron bar $[pt]$. If $pq \perp qb$, calculate $|pt|$ correct to one place of decimals, given that $|qb| = 7$ m and $|\angle pbq| = 60°$ as in diagram.



(50 marks)

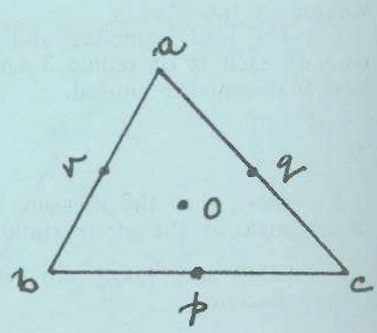
8. (a) If $\vec{a} = 4\vec{i} - 3\vec{j}$ and $\vec{b} = 5(\vec{i} + \vec{j})$ express $(\vec{b} - \vec{a})$ in terms of $\vec{i}$ and $\vec{j}$ and calculate $|\vec{ab}|$.

If $\vec{a} + t\vec{b} = k\vec{i}$ where $t, k \in \mathbf{R}$, find t and k and illustrate your result by a diagram.

(b) p, q, r are the midpoints of the sides of the $\Delta\, abc$ as in diagram and the medians of the triangle meet at o. Taking o as origin express

(i) $\vec{q}$ in terms of $\vec{a}$ and $\vec{c}$

(ii) $\vec{p} + \vec{q}$ in terms of $\vec{a}$, $\vec{b}$, and $\vec{c}$.



If $poqk$ is a parallelogram, prove that k is in the line cr.

(50 marks)