

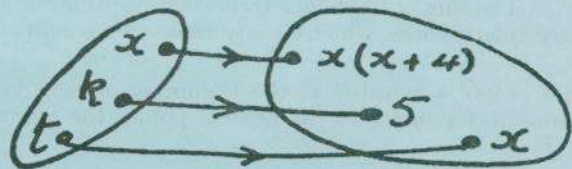
LEAVING CERTIFICATE EXAMINATION, 1978

MATHEMATICS—ORDINARY LEVEL—PAPER II (300 marks)

TUESDAY, 13 JUNE—MORNING, 9.30 to 12

Attempt **QUESTION 1** and **FOUR** other questions.

1. (i) Write the number $(9.75 \times 10^{-2})(8.04 \times 10^3)$ in the form $a \cdot 10^n$, where $1 \leq a < 10$ and $n \in \mathbf{Z}$.
- (ii) If $\log_{10} \left(1 + \frac{r}{100} \right) = 0.0792$, find r .
- (iii) If the two functions $f : x \rightarrow 5$ and $g : x \rightarrow x^2 - 3$ are defined on \mathbf{R} , find $fg(2)$ where fg is the composite function.
- (iv) Solve $\frac{1}{x-1} - \frac{1}{x} = \frac{1}{2}$
- (v) Find the range of values of $x \in \mathbf{R}$ for which $2x^2 + x > 6$
- (vi) Find the median of the data : 1.2, 2.5, 3.5, 3.1, 2.7, 1.5, 1.2, 3.1, 1.2
- (vii) If $z_1 = 2 + 3i$ and $z_2 = 3 + 2i$, evaluate $|z_1 z_2|$.
- (viii) Find the next term in the geometric sequence 1, $1\frac{1}{2}$, ...
- (ix) Evaluate the 3rd term in the expansion of $(1 - 2x)^{10}$ when $x = \frac{1}{2}$.
- (x) f is the function $x \rightarrow x(x+4)$ for $x > 0$
Find k .
If $t > 0$, express t in terms of x .
(See diagram)



(100 marks)

2. Find the point p on an Argand diagram which represents the complex number $3 - 4i$.
What complex numbers are represented by the image of p under the
- (i) axial symmetry in the real axis (i.e. x -axis),
(ii) projection on the imaginary axis (i.e. y -axis) parallel to the real axis,
(iii) central symmetry in the origin?
- If z_1, z_2, z_3 are the complex numbers represented by (i), (ii), (iii), above, respectively, evaluate

$$z_1 + 2z_2 + z_3 \quad \text{and} \quad \frac{z_1}{z_2} + \frac{z_3}{z_2}$$

(40 marks)

3. Solve the simultaneous equations:

$$\begin{aligned} x + y + z &= 1 \\ 2x + 3y + z &= 4 \\ 4x + 9y + z &= 16 \end{aligned}$$

and write down a fourth equation in x, y, z which is satisfied by your solution.

(40 marks)

4. The function $f : x \rightarrow x^3 - 3x^2 - 9x + 15$ is defined on $A = \{x \mid -2 \leq x \leq 5, x \in \mathbf{R}\}$.
Draw the graph of the function and use your graph to find, as accurately as you can,
- (i) the range of values of $x \in A$ for which $x > 0$ and $f(x) > 0$
(ii) the range of values of $x \in A$ for which $f(x) > 0$ and increasing
(iii) the values of $x \in A$, if any, which satisfy
 $x(x-3)(x+3) = 3(x-4)(x+4)$.

(50 marks)

[P.T.O.]

5. (a) Verify that 4 is the mean of the grouped frequency distribution:

CLASS INTERVAL	1 - 3	3 - 5	5 - 7	7 - 9
FREQUENCY	4	3	0	2

and calculate the standard deviation from the mean correct to one place of decimals.

(Note.—1-3 means 1 or greater but less than 3, etc. The data can be taken to be at the mid-interval values).

(b) In a random sample of shops the items h, k, r, s, t were bought by customers in the proportion 3 : 1 : 4 : 5 : 2, respectively. The prices of these items were noted on January 1st. The changes in THESE PRICES on February 1st and on March 1st are given :

ITEMS	h	k	r	s	t
CHANGES on FEB. 1	4p	10p	2p	6p	5p
CHANGES on MAR. 1	3p	9p	2p	7p	5p

On which date (Feb. 1 or Mar. 1) had the cost of living, as calculated on these items, increased the most?

(50 marks)

6. (a) Prove that the sum of the first n terms of an arithmetic series is $\frac{n}{2} (2a + (n-1)d)$, where a is the first term and d is the common difference.

The sum of the first 9 terms of an arithmetic series is 36 and the fifth term is 4. Prove that there are many arithmetic series which satisfy these two conditions and write down the first five terms of any one of them.

(b) £100 is invested at the beginning of each year for 5 consecutive years at 10% compound interest per annum. Calculate to the nearest pound the amount of this investment at the end of the fifth year.

(50 marks).

7. A farmer wants to plant wheat and potatoes but has to observe the following restrictions. He can give at most 2 day's labour per acre to wheat and at most 1 day's labour per acre to potatoes up to an overall maximum of 10 days. He cannot afford to spend more than £42 for labour at £7 per acre for wheat and £6 per acre for potatoes.

(i) Find the maximum acreage he can plant.

(ii) If the profit on wheat is £8 per acre and the profit on potatoes is £6 per acre, how many acres of each should he sow to maximize his profit?

(50 marks).

8. (a) Differentiate $3 - x^2$ with respect to x from first principles.

(b) Differentiate $2x^2(x^2 + 1)$ with respect to x and find the value of x for which the derivative is zero.

(c) Differentiate $\frac{2x^2}{x^2 + 1}$ with respect to x and find the coordinates of a point on the graph of $y = \frac{2x^2}{x^2 + 1}$ at which the tangent makes an angle of 45° with the positive sense of the x -axis.

(d) Given that the derivative of $\sin x$ is $\cos x$, find the derivative of $(\sin x)^3$.

(50 marks)