LEAVING CERTIFICATE EXAMINATION, 1977

MATHEMATICS—ORDINARY LEVEL—PAPER I

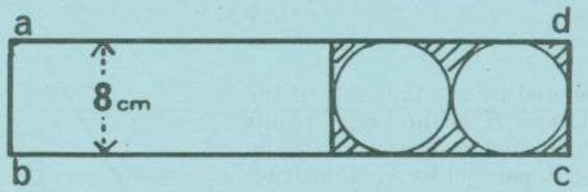
(300 marks)

MONDAY, 13 JUNE—MORNING, 9.30 to 12

Six questions to be answered.
All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.

1. (*a*) A circle is inscribed in a square so as to touch each of its four sides. What fraction of the area of the square is the area of the circle?
You may express your answer in terms of π.

(*b*) The top, bottom and curved side of a cylindrical can, whose diameter is equal in length to its height, are cut from a rectangular piece of tin as shown in the diagram (shaded region is waste)



Taking $\frac{22}{7}$ as an approximation for π and making no allowance for overlap, calculate

(i) the capacity of the can to the nearest cm^3,

(ii) the area of the rectange *abcd* to the nearest cm^2.

2. The points $a(0, -4)$ $b(6, 4)$ $c(-1, 3)$ are the vertices of a triangle.

(i) Show that the x-axis bisects $[ab]$.

(ii) Find the slope of the line bc and write down its equation.

(iii) Calculate the area of the $\triangle abc$.

(iv) Find the equation of the line through a which is perpendicular to the line bc.

3A. (i) Find the equation of the circle, centre the origin, which contains the point $(1, 7)$.
The point $(1, k)$ is inside the circle. If $k \in \mathbf{Z}$, find the least value of k.

(ii) The line $3x + y - 10 = 0$ is a tangent to the circle $x^2 + y^2 = 10$. Find the coordinates of the point of contact.

OR

3B. Take any line segment $[ab]$ and show clearly how to find the point $x \in [ab]$ such that

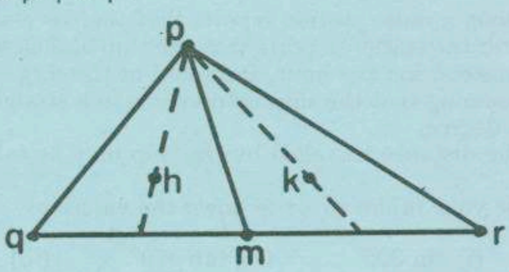
$$|ax|^2 = |ab| . |bx| .$$

If $|ab| = 2$, express in surd form the lengths $|ax|$ and $|bx|$ and then verify that

$$|ax|^2 = |ab| . |bx| .$$

4. Prove that the medians of a triangle are concurrent. Deduce that the centroid of a triangle (i.e. the point of concurrence of the medians) divides a median in the ratio 2 : 1.

pqr is a triangle in which $|qm| = 3 = |mr|$. If h and k are the centroids of the triangles *pqm* and *prm*, respectively, prove that $hk \parallel qr$ and calculate $|hk|$.



5. (*a*) o, a, b are three given points as in diagram. Construct

(i) the point p such that

$$\vec{op} = \tfrac{1}{2}(\vec{oa} + \vec{ob})$$

(ii) the point q such that

$$\vec{oq} = \vec{oa} - \vec{ob}.$$

If k is the point such that $\vec{ok} = \vec{op} - \vec{qo}$, prove that
$\vec{pa} = \tfrac{1}{2}\vec{pk}$.

(*b*) $\vec{i}$ and $\vec{j}$ are unit vectors along the x-axis and y-axis, respectively. If $\vec{u} = 3\vec{i} + 2\vec{j}$ and $\vec{w} = -\vec{i} + 4\vec{j}$, express in terms of $\vec{i}$ and $\vec{j}$ the vectors

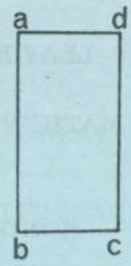
(i) $2\vec{w} + \vec{u}$ (ii) $\vec{wu}$.

If the point which represents the vector $t\vec{w} + \vec{u}$ is on the y-axis, find the value of the scalar t.

P.T.O.

6. *abcd* is a rectangle as in diagram and S_{ab} indicates the axial symmetry in the line *ab*. Construct the image of the line segment $[cd]$ under the composition of axial symmetries.

(i) $S_{ab} \circ S_{cd}$ (ii) $S_{ab} \circ S_{bc}$.

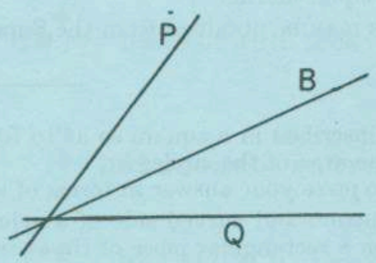


What single transformation is the same as the composition of two axial symmetries in perpendicular axes?
Prove your answer.

7. (*a*) P and Q are any two lines and B is the bisector of the angle between them as in diagram. What is the image of the line Q under

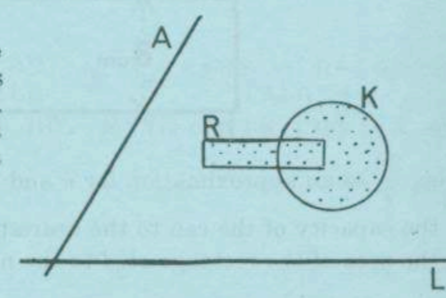
(i) $S_B \circ S_Q$ (ii) S_B,

where S_B and S_Q indicate the axial symmetries in B and in Q, respectively. Do you conclude from your answers to (i) and (ii) that $S_B \circ S_Q = S_B$? Give your reasons.



(*b*) R is the set of points enclosed by the four side of the rectangle (sides included) and K is the set of points enclosed by the circle (circle included) as in diagram. Let f be the projection on L parallel to A. Construct $f(R)$ and $f(K)$ and say, giving a reason, whether or not

$$f(R \cap K) = f(R) \cap f(K).$$



8. (*a*) Indicate clearly on your answer book the half-plane defined by

$$\{(x, y) \mid 2x + 3y \geqslant 18 \text{ for } x, y \in \mathbf{R}\}.$$

(*b*) A manufacturer uses small trucks and large trucks to transport his products in crates. Small trucks can take only 20 crates each while large trucks can take 30 crates each. Small trucks take 50 minutes each to load while large trucks take 90 minutes each to load. On a certain day at least 180 crates must be transported and on that day only 8 drivers, at most, are available. If the overall loading time is to be as short as possible, find how many of each type of truck should be used on that day.

	Small truck	Large truck
Crates	20	30
Loading time	50 min.	90 min.

9. At 12 noon a radar station reports that there is a ship 20 km away in the direction 10° West of North. One hour later the station reports that the ship is then 45 km away in the direction 25° East of North. Calculate, to the nearest km per hour, the speed of the ship.

Assuming that the ship is travelling in a straight line in the direction $x°$ North of East, calculate x to the nearest degree.

(The distance travelled by the ship may be taken to the nearest km.).

10. (*a*) Use your tables to write down the values of

(i) $\sin 300°$ (ii) $\tan 470°$ (iii) $\cos \frac{10\pi}{3}$.

(*b*) Using the same axes and the same scales sketch the graphs of the functions

$$x \to \cos x \text{ and } x \to \cos 2x$$

in the domain $0 \leqslant x \leqslant 2\pi$ for $x \in \mathbf{R}$.
Find from your graphs

(i) the values of x for which $\cos x = \cos 2x$
(ii) *one* value of x for which $\cos x = -\cos 2x$
(iii) the domain of x for which $\cos x \leqslant \cos 2x$.