AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1976

MATHEMATICS—ORDINARY LEVEL—PAPER II (300 marks)

MONDAY, 14 JUNE—MORNING, 9.30 to 12

Six questions to be answered.
All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.

1. A person invested £200 at $12\frac{1}{2}\%$ per annum compound interest. At the same time a second person invested £100 at 25% per annum compound interest. After how many years does the amount due to the second person begin to exceed the amount due to the first person?

2. When a is the first term and r is the common ratio of a geometric sequence, show how to establish that S_n, the sum to n terms, is

$$\frac{a(r^n - 1)}{r - 1}.$$

If s represents the nth term of the sequence, show that

$$S_n = \frac{sr - a}{r - 1}.$$

A ball rebounds from a floor to half the height from which it fell. After a number of bounces it rises to a height of 5/64 cm before being trapped on the floor.

The total distance the ball fell and rose between the first bounce and the time it was trapped on the floor was

$$\frac{2555}{32} \text{ cm}.$$

Find (i) the height of the first bounce,
(ii) the number of bounces until the ball was trapped.

3. (*a*) If $z_1 = 3 + 2i$ and $z_2 = 3 - i$, express $z_1^2 - z_2^2$ and $\frac{z_1}{z_2}$ in the form $x + iy$.

Verify that

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}.$$

(*b*) If $2 - i$ is a root of the equation $z^2 + tz + k = 0$, $t, k \in \mathbf{R}$ find the value of t and the value of k.

4A. How many subsets can be formed from the set $S = \{0, 1, 2\}$? List the subsets which
(i) include 0
(ii) include 1 but not 0.

Show that S is a group under addition (mod 3) but is not a group under multiplication (mod 3) assuming the associativity of the operations. Explain why $\{1, 2\}$ is a group under multiplication (mod 3).
Is $\{1\}$ a group under addition?
Is $\{1\}$ a group under multiplication?
Give reasons for your answers.

OR

4B. A factory produces 50 television sets in a particular week and tests in the factory gave rise to the following data:

number of faults per set	0	1	2	3	4	5	6
number of sets	1	8	12	11	9	5	4

What is the mean, $\bar{x}$, number of faults per set?
Calculate the standard deviation, σ, of the number of faults per set [See Tables Page 34].
From the data, estimate the percentage of sets that lie between $\bar{x} + \sigma$, and $\bar{x} - \sigma$.

5. (*a*) When $A = \{0, 1, 2, 3\}$ the function f is defined by

$$f : A \to \mathbf{R} : x \to 2^x.$$

Name the range (image) of f and say, giving a reason, whether f is an injection or a surjection.

(*b*) K is an order relation defined on the set $\{p, q, r\}$. Two couples of K are (p, p) and (p, q). How many relations K are possible? Draw the graph in each case.

(*c*) $g(x)$, $x > 0$, $x \in \mathbf{Z}$, is defined as the least integer greater than $\sqrt{x}$. What is the value of $g(4)$? Find the values of x for which $g(x) = 4$.

6. Let f be the function defined by

$$f : \mathbf{R} \to \mathbf{R} : x \to 2x + 2.$$

Name the function $g : \mathbf{R} \to \mathbf{R}$ so that $g = f^{-1}$ (i.e. the inverse of f)

Using the same axes and scales plot the graphs of the functions f and g in the domain $-2 \leqslant x \leqslant 2$. Draw in the line L such that g is the image of f under the axial symmetry in L.

Find the functions

(i) $q : \mathbf{R} \to \mathbf{R} : x \to f \circ f(x)$

(ii) $s : \mathbf{R} \to \mathbf{R} : x \to g \circ g(x)$.

Using the same axes and scales as before, plot the graphs of the functions q and s in the domain $-2 \leqslant x \leqslant 2$.

Draw in the line M such that s is the image of q by the axial symmetry in M. What is the relation between the functions q and s?

7. Fill in the empty spaces in the following table for values of $f : \mathbf{R} \to \mathbf{R} : x \to 16x^3 - 20x^2 - 12x + 9$.

x	-1	$-0{\cdot}5$	0	$0{\cdot}25$	$0{\cdot}75$	1	$1{\cdot}25$	$1{\cdot}75$
$f(x)$		$8{\cdot}0$		$5{\cdot}0$	$-4{\cdot}5$		$-6{\cdot}0$	$12{\cdot}5$

Draw the graph of the function in the domain $-1 \leqslant x \leqslant 1{\cdot}75$ using the scales 10 cm = 1 on the x-axis and 2 cm = 5 on the y-axis. From the graph, write down:

(i) the values of x when $f(x) = 0$,

(ii) the domain of values of x for which the gradient (slope) of the graph is negative,

(iii) the range of values of k for which $f(x) = k$ has three real roots,

(iv) the values of l for which $f(x) = l$ has two real roots,

(v) the values of x for which $f(x) = 10x$.

8. (*a*) Verify that ${}^{10}C_7 = {}^{10}C_3$. Hence, or otherwise, calculate the value of ${}^{20}C_{17}$

(*b*) Write out the binomial expansion of $(1 + x^2)^5$ and find the exact value of the coefficient of the term containing x^8.

Using the binomial expansion, find the value of $(1{\cdot}04)^5$, correct to four places of decimals.

9. (*a*) Differentiate $7 - 3x^2$ from first principles. If the rate of change of $7 - 3x^2$ with respect to x is -6, find the value of x.

(*b*) Find the coordinates of the points of the curve

$$f : \mathbf{R} \to \mathbf{R} : x \to \frac{2 - x}{x^2 - 3}$$

where the tangents to the curve are parallel to the x-axis.

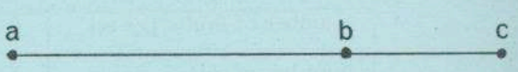
10. (*a*) Evaluate

(i) $\displaystyle\int_{-1}^{5} dx$; (ii) $\displaystyle\int_{2}^{3} x(2 - x)\, dx$; (iii) $\displaystyle\int_{0}^{1} (3 - x)^2\, dx$.

(*b*) The speed of a car travelling from a to c is given by

$$\frac{ds}{dt} = 5(1 + t^2)$$

where s is its distance in metres from a and t is the time in seconds measured from a.



(i) Find the speed of the car as it passes through a.

(ii) When $t = 3$, the car is at b. When $t = 4$ the car is at c. Find $|\, bc \,|$.