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LEAVING CERTIFICATE EXAMINATION, 1975

MATHEMATICS—ORDINARY LEVEL—PAPER II (300 marks)

MONDAY, 16 JUNE—MORNING, 9.30 to 12

Six questions to be answered.
All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.

1. If the interest rate is $r\%$, £1 will become £$\left(1 + \frac{r}{100}\right)$ after one year. What will £1 become after two years at (i) simple interest (ii) compound interest?
What sum of money will become £1 after two years at $r\%$ per annum compound interest?
A loan of £100 at 20% per annum, compound interest, is to be repaid in full by two equal annual instalments, due at the end of each year. Calculate, to the nearest penny, the amount of each repayment.

2. (*a*) Prove that n^2 is the sum of the first n odd natural numbers.
If the odd numbers are arranged as follows:
$$\{1\};\quad \{3, 5\};\quad \{7, 9, 11\};\quad \{13, 15, 17, 19\};\ \ldots\ldots\ldots$$
and
$$O_1 = \{1\}$$
$$O_2 = \{1, 3, 5\}$$
$$O_3 = \{1, 3, 5, 7, 9, 11\}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots,$$
list the elements of O_4 according to this pattern.
Find (i) the number of elements in O_{20}
(ii) the sum of all the numbers in O_{20}.
(*b*) What is the sum to infinity of the geometric series
$$a + ar + ar^2 + ar^3 + \ldots,\ |r| < 1?$$
Find the geometric series whose third term is 4, such that its sum to infinity is three times its first term.

3. (*a*) Solve the equation $z^2 - 4z + 13 = 0,\ z \in C$.
If z_1, z_2 are the roots of the equation, plot and indicate clearly, the points on an Argand diagram representing:
$$z_1;\qquad z_2;\qquad z_1 + z_2.$$
Show that
(i) $|z_1 + z_2| < 2\,|z_1|$ (ii) $|z_1 z_2| = |z_1|^2$.
(*b*) Find $x, y \in R$ such that $(4 + 3i)(x + yi) = 1 + 7i$.

4A. If X, Y are the axes of symmetry, and o is the centre of symmetry of a rectangle *abcd* which is not a square, construct Cayley tables for the composition of
(i) rotations about o which map the rectangle *abcd* onto itself
(ii) axial symmetries in X, Y.
Assuming the associativity of composition, show that one of the Cayley tables forms a group.
Investigate the possibility that the union of the sets in (i) and (ii) forms a commutative group of order 4.

OR

4B. The following frequency table gives the results of one hundred candidates at an examination:

Marks	0–20	20–40	40–60	60–80	80–100
Candidates	5	15	40	30	10

where (0–20) means 0 but less than 20, (20–40) 20 but less than 40 etc.
Calculate $\bar{x}$, the mean of the distribution.
Draw a cumulative frequency curve to illustrate the data.
If $\sigma = 20$ is the standard deviation, estimate from your graph the percentage of candidates who got marks between $\bar{x} - \sigma$ and $\bar{x} + \sigma$, inclusive.

[P.T.O.

5. (a) T and D are two relations on the set $S = \{2, 3, 4, 11, 12\}$ such that when $x, y \in S$

$$T = \{(x, y) \mid x = y \pmod 3\}$$
$$D = \{(x, y) \mid x \text{ divides } y\}.$$

Illustrate the relations graphically and say, giving reasons, whether or not each relation is (i) reflexive (ii) symmetric (iii) transitive.

(b) If $A = \{a, b, c\}$, $B = \{p, q, r, s\}$, say which of the following relations $A \to B$ are functions: $R_1 = \{(a, p), (a, q), (b, r), (c, s)\}$; $R_2 = \{(a, s), (b, s), (c, s)\}$; $R_3 = \{(a, p), (b, r), (c, s)\}$.

In the case of each function state the range and say, giving reasons, whether or not it is (i) injective (ii) surjective.

6. The functions f and g are defined as follows:

$$f : R \to R : x \to x + 2$$
$$g : R \to R : x \to 4x^2 - 1.$$

Establish whether the statement

$$g(a + b) = g(a) + g(b)$$

is true or false when (i) $a = b = 0$ (ii) $a = \frac{1}{2}, b = -\frac{1}{4}$.

Evaluate $f(-2)$, $g(-2)$, $g \circ f(-2)$ and determine the coefficients k, m, so that

$$g \circ f(x) = g(x) + kf(x) + m, \qquad \text{for all } x \in R.$$

Express $f \circ g(x)$ and $g \circ f(x)$ in terms of x and find the solution set of the inequality

$$f \circ g(x) \leqslant g \circ f(x).$$

7. (a) Solve the equation $x^3 - 3x^2 - 4x + 12 = 0, \quad x \in Z$.

(b) Draw the graph of the function

$$f : R \to R : x \to \tfrac{1}{6}(x^3 - 3x^2 - 4x - 6)$$

for values of x in the domain $-2 \leqslant x \leqslant 5$.

Find from your graph

(i) the value of x for which $f(x) = 0$

(ii) the domain of f for which the function is negative and decreasing

(iii) the roots of the equation $x^3 - 3x^2 - 4x + 6 = 0$.

8. (a) Find the number of subsets of five letters that can be formed from the following set

$$\{e, d, u, c, a, t, i, o, n\}.$$

How many of these subsets

(i) contain the letter t (ii) do not contain the letter d?

(b) Write out the Binomial expansion of $(1 + x)^6$. Use this expansion to evaluate $(10{\cdot}2)^6$, correct to five significant figures.

9. (a) Differentiate $4x - x^2$ from first principles.

Find the point on the curve $y = 4x - x^2$ at which the tangent to the curve makes an angle measuring 45° with the positive direction of the x-axis.

(b) Find the derivative of the function

$$f : x \to \frac{5 + 4x - x^2}{4x - x^2}, \qquad 0 < x < 4.$$

Verify that the tangent to the curve $y = f(x)$ at the point $(2, 9/4)$ is parallel to the x-axis.

10. (a) Evaluate

$$\text{(i)} \int_{-2}^{-1} 3dx \qquad \text{(ii)} \int_{0}^{1} (1 - x^2)\, dx \qquad \text{(iii)} \int_{2}^{3} (4x - x^2)\, dx.$$

(b) The volume V of water in a small reservoir is increasing at a rate given by

$$\frac{dV}{dt} = 30t + t^2$$

where t is measured in days and V in cubic metres. Calculate the increase in the volume of the water from the end of the first day to the end of the second day.