LEAVING CERTIFICATE EXAMINATION, 1975

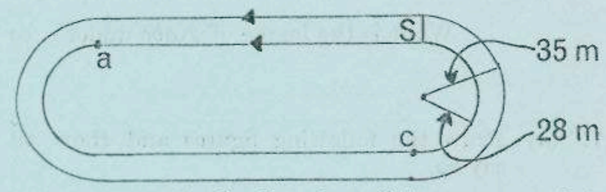
MATHEMATICS—ORDINARY LEVEL—PAPER I
(300 marks)

WEDNESDAY, 11 JUNE—MORNING, 9.45 to 12.15

Six questions to be answered.
All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.

1. The diagram represents the actual paths taken by two runners, each path consisting of two straight runs and two semi-circular runs at each end of radii 28 m and 35 m, respectively.

Taking 22/7 for π, find the length of one straight run in order that one complete lap of the inside path is 440 m.



S represents the finishing line in a 440 m race. If the runners go in the direction of the arrows and the inside runner starts from the line S, how far from S on the outside path should the outside runner start?

If the inside runner wants to complete the race in $1\frac{1}{2}$ mins at a constant pace what times (in seconds) should show on a stop-watch at points (i) a (ii) c?

2. (*a*) The extremities of the diameter of a circle are the points (3, 4) and (− 3, − 4). Find
   (i) the centre of the circle
   (ii) the length of its radius.
   Write down the equation of the circle.

   (*b*) The points a (9, 8), b(1, 4) and c(3, 0) are the vertices of a triangle abc. Prove $ab \perp bc$ and find the area of the triangle abc.

3A. (*a*) Write down the lengths of the radii of the two circles
   (i) $x^2 + y^2 = 13$
   (ii) $5x^2 + 5y^2 = 66$.

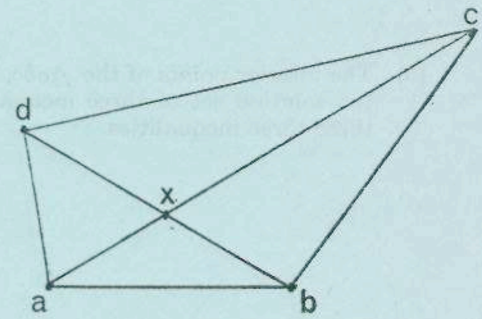
   (*b*) Find the coordinates of the points at which the line K, $x + 2y - 5 = 0$, cuts the axes and draw K on a diagram. Find the equation of the line Q through (0, 0) perpendicular to K and prove that the circle $x^2 + y^2 = 5$ passes through the point of intersection of K and Q.

or

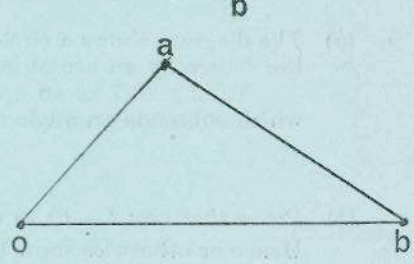
3B. $[ab]$ is a line segment. Construct a point $x \in [ab]$ such that $|ab| \,.\, |bx| = |ax|^2$. Explain your construction.
   If $|ab| = 1$, $y \in [ab]$ and $|ba| \cdot |ay| = |by|^2$, show that
   (i) $|by| = \frac{1}{2}(\sqrt{5} - 1)$
   (ii) $|xy| = \sqrt{5} - 2$.

4. Prove that the perpendicular bisectors of the sides of a triangle meet in a point (the circumcentre).

The diagram shows a quadrilateral $abcd$. The intersection of the diagonals is x. Prove that the circumcentres of the four triangles xcd, xda, xba and xbc are the vertices of a parallelogram.

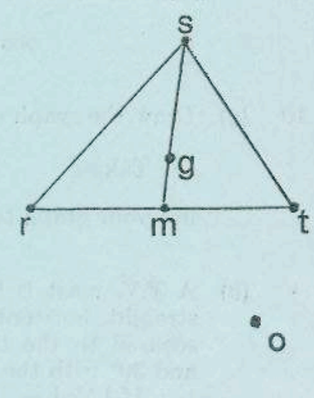


5. (*a*) Draw a sketch of $\triangle abo$, as shown, and on your diagram
   (i) find the position of k such that
   $$\text{vector } \vec{ok} = \tfrac{1}{2}(\vec{oa} + \vec{ob})$$
   (ii) find the position of t such that vector $\vec{ot} = \frac{1}{2}(\vec{oa} - \vec{ob})$.



   (*b*) rst is a triangle as shown, where m is the mid-point of $[rt]$ and $g \in [ms]$ such that
   $$|mg| = \tfrac{1}{3}\,|ms|$$
   Given also a point o as shown. Write the vector $\vec{ms}$ in terms of the vectors $\vec{rs}$ and $\vec{rt}$ and hence write the vector $\vec{rg}$ in terms of the vectors $\vec{rs}$ and $\vec{rt}$.

   Deduce that $\vec{og} = \frac{1}{3}(\vec{or} + \vec{os} + \vec{ot})$.

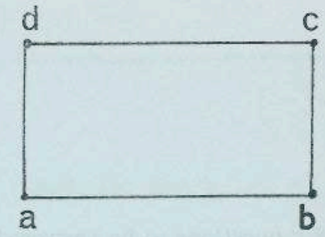


6. (*a*) Prove that the composition of two central symmetries is a translation.

abcd is a rectangle as shown.

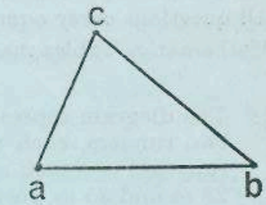
Write down as translations $S_b \circ S_a$ and $S_d \circ S_c$ and find the image of the rectangle under

$$(S_d \circ S_c) \circ (S_b \circ S_a).$$



(*b*) *abc* is a triangle, as shown. Name a single translation which equals $\vec{bc} + \vec{ab}$.

What is the image of $\triangle abc$ under $\vec{ca} + \vec{bc} + \vec{ab}$?



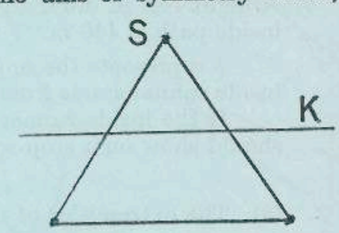
7. (*a*) Draw the following figures and show all their axes of symmetry. If no axis of symmetry exists, say so.

(i) a square

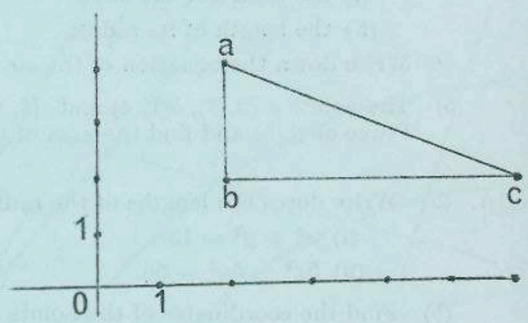
(ii) a semi-circle.

(iii) a parallelogram (not a rectangle or a rhombus)

S is an equilateral triangle and *K* is a line through the midpoints of two of its sides (see diagram). Construct a set *T* such that *K* is an axis of symmetry of $T \cup S$.



(*b*) The coordinates of *a*, *b* and *c* are (2, 4), (2, 2) and (6, 2), respectively, and *o* is the origin. If the image of *b* is (– 2, 2) under a certain rotation of centre *o* find the images of *a* and *c* under that rotation.



8. (*a*) Using *separate* diagrams indicate clearly the half planes defined by the following

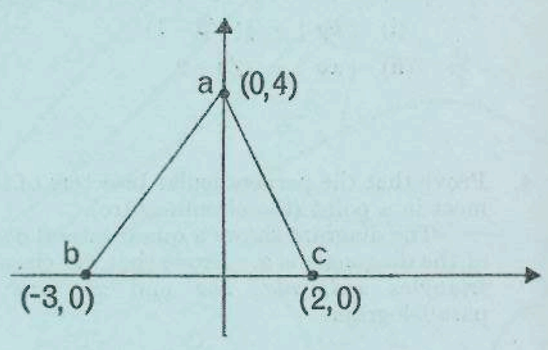
(i) $\{(x, y) \mid x \geqslant 0, x \in R, y \in R\}$

(ii) $\{(x, y) \mid y \leqslant -4, x \in R, y \in R\}$

(iii) $\{(x, y) \mid 3x + y \leqslant 6, x \in R, y \in R\}$.

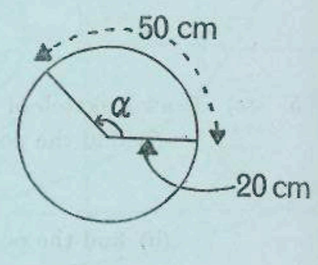
In each case give the coordinates of one point, not on the boundary of the half-plane, which satisfies the given inequality.

(*b*) The interior points of the $\triangle abc$, as shown, represent the solution set of three inequalities. Write down these three inequalities.



9. (*a*) The diagram shows a circle of radius 20 cm and α is an angle subtended at the centre by an arc of length 50 cm. Find the measure of α in radians.

Taking 22/7 as an approximate value of π, find the length of an arc which subtends an angle of 210° at the centre (answer to nearest cm).



(*b*) Prove that $\cos(A - B) = \cos A \cos B + \sin A \sin B$.

Hence or otherwise show that

$$\cos 15° = \frac{1}{2\sqrt{2}}(1 + \sqrt{3})$$

10. (*a*) Draw the graph of the function $x \to \sin x$ in the domain $0 \leqslant x \leqslant 2\pi$.

Taking $\sin \frac{3\pi}{10} = 0{\cdot}8090$,

use your graph to find the domain of values of x for which

$$-0{\cdot}8090 \leqslant \sin x \leqslant 0{\cdot}8090$$

(*b*) A T.V. mast is held in a vertical position on top of a straight horizontal wall by wires, two of which are secured to the top of the mast making angles of 65° and 30° with the top of the wall, as shown.

If $|hk| = 15$ m, find, to the nearest metre, the length of the longer wire.