L 34 **AN ROINN OIDEACHAIS** M. 48

LEAVING CERTIFICATE EXAMINATION, 1974

MATHEMATICS—ORDINARY LEVEL—PAPER II (300 marks)

TUESDAY, 18 JUNE—MORNING, 9.30 to 12

Six questions to be answered.
All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.

1. Calculate, to the nearest pound, the compound interest on £185 for 10 years at 12% per annum.
£2000 is borrowed at 8% per annum. £800 is repaid at the end of the first year and the same amount at the end of the second year. Calculate, to the nearest penny, the amount which must be paid at the end of the third year to clear the debt.

2. (*a*) $\left(2n - \frac{3}{2}\right)$ is the *n*th term of a sequence. Write down its first three terms and verify that they form an arithmetic sequence. Assuming that the sequence is arithmetic, find its sum to *n* terms. What is the least number of terms which must be taken so that the sum exceeds 60?
(*b*) What is the sum to *n* terms of the geometric series
$$a + ar + ar^2 + \ldots\ldots ?$$
What is the condition that must exist if the series can be summed to infinity? Assuming this condition, what is the sum to infinity of the series? Hence, or otherwise, write $0\cdot\dot{3}\dot{1}$ in the form
$$\frac{p}{q},\ (p, q \in Z, q \neq 0).$$

3. (*a*) Solve the equation $z^2 + 4z + 29 = 0$, where $z \in C$, the set of complex numbers, and represent the roots on an Argand diagram.
(*b*) If $z_1 = 2 - 3i$ and $z_2 = -3 + i$, find $|z_1|$, $|z_2|$, $|z_1 - z_2|$.
Write down three other complex numbers which have the same modulus as z_1.
(*c*) If $z_4 = 3 + 2i$, $z_5 = 2i$, $z_6 = -2 - i$, verify that $(z_4 \times z_5) \times z_6 = z_4 \times (z_5 \times z_6)$.

4A. (*a*) If O is the set of odd and E is the set of even natural numbers, and if N, Z, R are the sets of natural numbers, integers and rationals, respectively, name **one** of these sets in each case which
(i) is not closed under addition
(ii) has no identity element for multiplication
(iii) has one, and only one element, which has no inverse under multiplication
(iv) has more than one element which has no inverse under multiplication.
(*b*) Show that the set of rotations of an equilateral triangle about its centroid which maps the triangle onto itself forms a group under composition. (You may assume associativity).

or

4B. There are 49 students in a class participating in a savings scheme. The table shows the distribution of the money in the scheme on a certain date:

Amount of money in £	0–2	2–4	4–6	6–8	8–10	10–12	12–14
No. of students	3	5	9	14	10	6	2

[Note: 0–2 means 0 and less than 2, 2–4 means 2 and less than 4, etc.]

Draw a histogram to represent the distribution. Use the histogram to estimate the mode of the distribution. Fill in the cumulative frequency table below and use it to draw a cumulative frequence polygon of the distribution.

Amount of money in £	< 2	< 4	< 6	< 8	< 10	< 12	< 14
No. of students							

Hence estimate the median of the distribution.

5. Give an example to illustrate each of the following:
   (i) a relation which is not a function
   (ii) a relation which is a function
   (iii) a function $f : R \to R$
   (iv) a function $g : R \to R$ which is an injection.

Let $B = \{t \mid -1 \leqslant t \leqslant 1, t \in R\}$ and let k be the function $R \to B$: $x \to \sin x$. Say why k is a surjection.
If f is a function $x \to f(x)$, explain the difference between f and $f(x)$.
h is the function defined by

$$h : R_0 \to R_0 : x \to \frac{1}{x},$$

where R_0 is the set of real numbers excluding 0.
Find (i) $h(2)$ (ii) $h(\frac{1}{2})$.
Establish whether or not

$$h(2 + \tfrac{1}{2}) = h(2) + h(\tfrac{1}{2}).$$

6. (*a*) $R_1 = \{(a, b), (b, c), (c, d), (d, a), (e, b)\}$
$R_2 = \{(a, c), (b, d), (c, a), (d, b), (e, e)\}$.
Write out the domain and range of R_1.
Is R_1 transitive? Why?
Is R_2 reflexive? Why?
Is $R_1 \circ R_2 = R_2 \circ R_1$? Give your reason.
Write out the couples of the inverse of $(R_1 \circ R_2)$. Is this inverse relation the same as $R_1^{-1} \circ R_2^{-1}$? Why?
(*b*) If $x \in \{-4, -3, -2, -1, 0, 1, 2, 3\}$, write down the sets of values of x for which

(i) $1 + 3x \geqslant 4x + 3$ (ii) $\frac{1}{x} \leqslant \frac{1}{2}$ (iii) $x^2 \leqslant 4$.

7. (*a*) Solve the equation $2x^3 - 3x^2 - 8x - 3 = 0$.
(*b*) Draw the graph of the function $f : x \to x^3 - 3x^2 - 2x + 5$ in the domain $-2 \leqslant x \leqslant 4$.
Find from the graph (i) the roots of $f(x) = 0$
(ii) the values of x for which $f(x) = 4$,
(iii) the domain of values of x for which f is negative and decreasing.

8. (*a*) Show ${}^6C_3 + {}^6C_2 = {}^7C_3$.
(*b*) Write out the expansion of $(a - 2b)^6$.
If $a = 1$, $b = \frac{1}{100}$, find which term of the expansion has 4 zeros immediately after the decimal point.
Use the expansion to evaluate $(0{\cdot}98)^6$ correct to four decimal places.

9. (*a*) Prove $\dfrac{d\left(\dfrac{u}{v}\right)}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$.
Find the derivative with respect to x of $\dfrac{x^2 + 2}{1 - 3x}$, when $x \neq \frac{1}{3}$.
What is the slope of the tangent to the curve $y = \dfrac{x^2 + 2}{1 - 3x}$ at $x = 2$?
(*b*) Evaluate (i) $\displaystyle\int_{-1}^{1} dt$ (ii) $\displaystyle\int_{1}^{3} 2(x^2 - 1)\, dx$.

10. (*a*) Differentiate from first principles $5 + 2x - 5x^2$ with respect to x.
(*b*) A missile is fired vertically upwards and its height h, in metres, above the ground after t seconds is $500 + 200t - 5t^2$.
   (i) If t was measured from the time the missile was fired, how high above the ground was the missile when it was fired?
   (ii) If the missile's speed is given by $\dfrac{dh}{dt}$, find its speed after 3 seconds.
   (iii) After how many seconds does the missile just begin to fall back downwards? How far above the ground is it at this time?