LEAVING CERTIFICATE EXAMINATION, 1974

MATHEMATICS — ORDINARY LEVEL — PAPER I
(300 marks)

MONDAY, 17 JUNE—MORNING, 9.30 to 12.

Six questions to be answered.
All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.

1. Taking 22/7 as an approximation for π, calculate the volume of a cylinder of height 7 cm and of radius 1 cm.
   Water flows through a pipe of internal diameter 2 cm at a rate of 7 cm per second into an empty rectangular tank that is 1·2 m long, 1·1 m wide and 30 cm high. How long will it take to fill the tank?
   If the diameter of the pipe had been 1 cm, at what rate should the water flow in order to fill the tank in the same time?

2. Find the coordinates of the points where the line $y = x + 3$ cuts the axes and draw a sketch of the line.
   Reflect this line in the x-axis and find the equation of its image.
   Prove that the angle between the line and its image is a right angle.
   Find the area of the triangle formed by the line, its image and the y-axis.

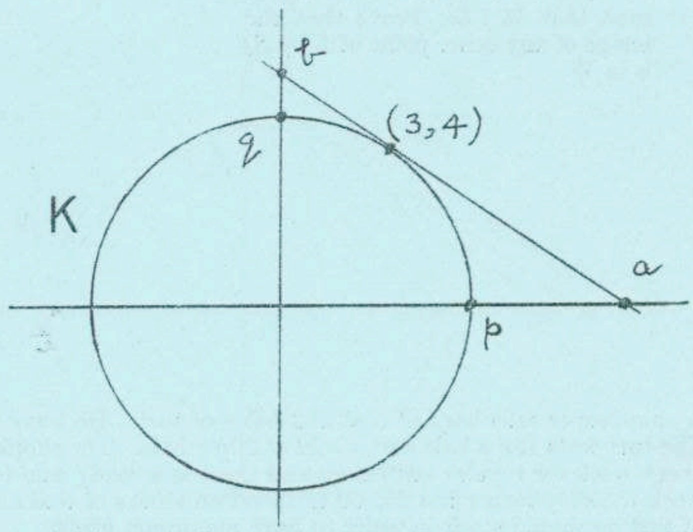
3A. $x^2 + y^2 = 25$ is the equation of the circle K and ab is the tangent to K at the point (3, 4). See diagram.
Verify that

   (i) (3, 4) is a point of K,

   (ii) $(-\frac{3}{4})$ is the gradient of ab,

   (iii) $3x + 4y - 25 = 0$ is the equation of ab,

   (iv) $|ab|$ is greater than the length of the quarter arc pq.



OR

3B. Show, with proof, how to construct an isosceles triangle having each of the angles at the base double the angle at the vertex.

4. abc and def are two triangles in which $\angle bac = \angle edf$ in measure and $\angle abc = \angle def$ in measure. Prove that

$$\frac{|ab|}{|de|} = \frac{|bc|}{|ef|}.$$

   $wxyz$ is a parallelogram and r is a point of the side $[wz]$ such that $|wr| = 2\,|rz|$. The diagonal $[wy]$ cuts $[xr]$ in the point s. Prove

   (i) $|xy| = \frac{3}{2}\,|wr|$, (ii) $|xs| = \frac{3}{2}\,|sr|$.

5. $\vec{i}$ and $\vec{j}$ are unit vectors along the x-axis and y-axis, respectively, and $o(0, 0)$ is the origin. If a (3, 4) and $b(7, 7)$ are two points, write the vectors $\vec{oa}$ and $\vec{ab}$ in terms of $\vec{i}$ and $\vec{j}$ and verify that $\vec{oa} + \vec{ab} = \vec{ob}$.
   Calculate $|\vec{oa}|$, $|\vec{ab}|$ and $|\vec{oa} + \vec{ab}|$ and verify that

$$|\vec{oa} + \vec{ab}| < |\vec{oa}| + |\vec{ab}|.$$

   If c is the point (13, 15), investigate if the vector $(\vec{ob} - \vec{oc})$ is collinear with $\vec{oa}$.

[P.T.O.

6. Prove that the composition of two axial symmetries (reflections) in parallel axes which are disjoint is a translation.

Deduce that the composition of three axial symmetries in parallel axes which are disjoint is an axial symmetry.

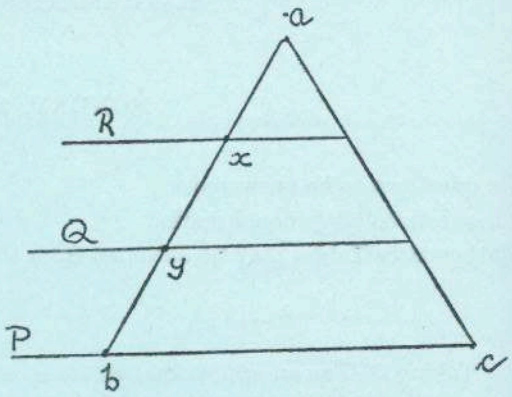
abc is an equilateral triangle. P, Q, R are parallel lines cutting $[ab]$ in b, y and x such that

$$|\, ax \,| = |\, xy \,| = |\, yb \,|$$; see diagram.

Where is the line T such that

$$S_R \circ S_Q \circ S_P = S_T$$

and construct the image of the $\triangle\, abc$ by $S_R \circ S_Q \circ S_P$.



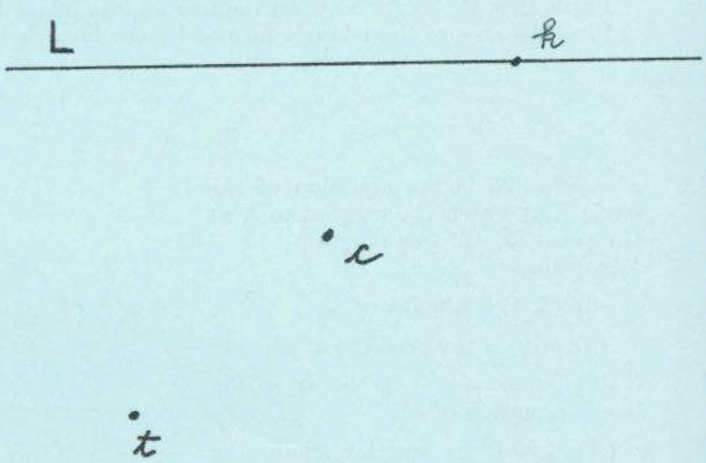
7. (*a*) Explain, with the aid of a diagram, the meaning of "parallel projection of the plane on a line L parallel to a line K".

Say, giving a reason, why a parallel projection is a function and why the inverse of a parallel projection is not a function.

The images of two couples (a, b) and (c, d) by a parallel projection are (p, q) and (r, s), respectively. Does $(p, q) \uparrow (r, s) \Rightarrow (a, b) \uparrow (c, d)$? Illustrate your answer.

(*b*) L is a line and c is a point. $k \in L$ and t is the image of k by the central symmetry S_c (see diagram).

Let W be the line through t such that $W \parallel L$. Prove that the image of any other point of L by S_c is in W.



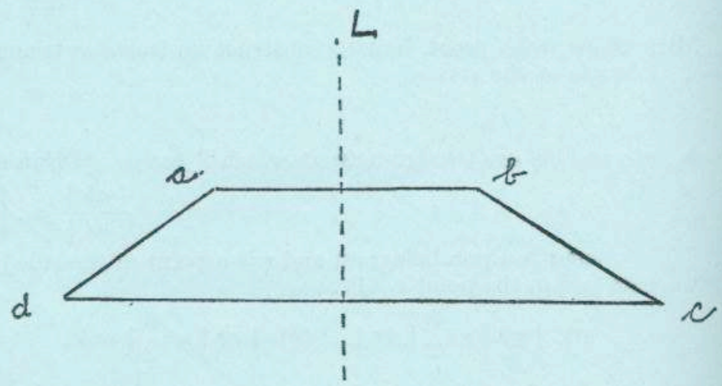
8. A shopkeeper sells bags of coal and bales of turf. He buys the coal at 45p a bag and sells it at 59p a bag. The turf costs 15p a bale and is sold at 20p a bale. The shopkeeper needs 50 bags of coal and 50 bales of turf every week for regular customers and there is a ready sale for all the coal and turf he stocks. In a certain week the shopkeeper has £52·50 to spend on stocks of coal and turf. Find how many bags of coal and bales of turf he should stock in order to have maximum profit.

9. The roof of a house has the shape *abcd*, as in diagram, and the shape is symmetrical about the broken line L. If $|\, cd \,| = 60$ m, the measure of $\angle adc = 30°$ and the parallel sides $[ab]$ and $[dc]$ are 6 m apart, calculate

(i) the area of *abcd*, correct to the nearest square metre,

(ii) the length $|\, bd \,|$, correct to the nearest metre.

[See Tables p. 20 to p. 25]



10. (*a*) Write down the period and range of each of the functions

(i) $x \rightarrow \cos x$ (ii) $x \rightarrow 2\cos 2x$ (iii) $x \rightarrow \frac{1}{2}\cos\frac{1}{2}x$.

If $\cos ax = \cos a(x + l)$, $a > 0$, $l > 0$, write down the least positive value of al in terms of π. Find the value of a for which $l = 3\pi$ and hence, or otherwise, write down a cosine function that has period 3π and range $(-3, 3)$.

(*b*) Using the same axes and the same scales sketch the functions

$$x \rightarrow \sin x \text{ and } x \rightarrow \sin 2x$$

in the domain $0 \leqslant x \leqslant 2\pi$.

Use your graph to write down the values of x for which $\sin x = \sin 2x$ in the given domain.