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LEAVING CERTIFICATE EXAMINATION, 1973

MATHEMATICS—ORDINARY LEVEL—PAPER II (300 marks)

WEDNESDAY, 13 JUNE—MORNING, 9.30 to 12

Six questions to be answered.
All questions are of equal value.
Mathematics Tables may be obtained from the Superintendent.

1. Find, to the nearest penny, the compound interest on £200 for 3 years at $4\frac{1}{2}\%$ per annum.
   Find the least number of years it will take £200 to amount to £370 at $4\frac{1}{2}\%$ per annum compound interest.

2. (a) "A sequence is a function $f: N_0 \to R: n \to f(n)$." Explain this statement and give an example to illustrate your explanation.
   $1/3^{n+1}$ is the nth term of a sequence. Write down the first three terms of the sequence and find S_n, the sum to n terms.
   What limit does S_n approach as n tends to infinity?

   (b) In his first year a man received £x in salary. Each year after that he received an addition of £y to the preceding year's salary. If his salary in his 8th year was £1,330 and in his 17th year was £1,915, find the value of x and the value of y.

3. (a) If $z = 2 - 3i$, plot on an Argand diagram:
   (i) $2z$, (ii) $-3z$, (iii) iz, (iv) z^2.

   (b) Write $\dfrac{-1 + 3i}{4 - 3i}$ in the form $x + iy$, where $x, y \in R$.

   (c) Solve for x and y the equation:
   $$(x + iy) + (3 - i) = 2(1 - 3i) - (y - ix).$$
   where $x, y \in R$.

4A. (a) By drawing up a Cayley Table, or otherwise, prove that the set $\{1, -1, i, -i\}$ under the binary operation of multiplication is a group. $(i = \sqrt{-1})$. (You may assume that multiplication of complex numbers is associative)
   Is the group commutative? Give your reason.

   (b) $(\{0, 1, 2\}, + \pmod 3)$ is a group.
   (i) Write down the identity element.
   (ii) State in each case the inverse of 0, 1, 2.
   (iii) Solve the equation $2x - 5 = 2 \pmod 3$.
   (Note $2x$ means $x + x$).

   or

4B. (a) Calculate the mean of the array of numbers: 7, 9, 11, 12, 16, 16, 20.

   (b) The intake of milk (in million gallons) in a one year period by the 37 branches of a set of creameries is shown in the following table:

Million Gallons	0–1	1–2	2–3	3–4	4–5	5–6
Number of Branches	3	14	10	6	3	1

   [Note: 0–1 means 0 but less than 1, 1–2 means 1 but less than 2 etc.]
   Draw a histogram to illustrate this distribution. Use the histogram to estimate the median of the distribution.

5. (a) $A = \{9, 3, 1, \frac{1}{3}\}$ and f is a function defined on A given by
   $$f: A \to R: x \to \log_3 x.$$
   If B is the range (image) of f, list the elements of B and say, giving a reason, whether f is an injection or a surjection.

   (b) $M = \{x \mid 2 \leqslant x \leqslant 9, x \in N\}$ and P is a relation defined on M given by
   $$P = \{(x. y) \mid x \text{ divides } y\}.$$
   Write out the elements of P.

   (c) E is an equivalence relation defined on the set $\{a, b, c\}$ and (a, a), (a, b), (b, c) are three elements of E. List the other elements of E.

6. (a) f and g are functions defined as follows:

$$f: x \to x + 3$$
$$g: x \to 2x^2 + 1.$$

Find $f(-1)$, $g(2)$, $f(g(2))$, $g(f(-1))$.
Solve $g(f(x)) = 9$.

(b) Find the solution set of

$$\{x \mid 2x^2 + 7x + 8 = 0, x \in C\},$$

where C is the set of complex numbers.

(c) Find the maximum value of the function

$$x \to 3 - (x^2 + 5x)$$

and find the value of x that gives this maximum.

7. (a) Find the factors of $4x^3 + 8x^2 - x - 2$.

(b) Draw the graph of the function

$$f: R \to R: x \to 2 + 4x - x^3 = f(x)$$

in the domain $-3 \leqslant x \leqslant 3$.
Find from the graph, as accurately as you can,
(i) the roots of $f(x) = 0$,
(ii) that part of the domain for which $f(x)$ is positive and increasing.

8. (a) Write down the value of $^8C_3$ and show that $^8C_3 = {}^8C_5$.

(b) Is $8! = 3! \times 5!$? Give your reason.

(c) Use the Binomial Theorem to write down the first four terms of the expansion $f(1 + 3x)^8$. Hence, or otherwise, find the value of $(1 \cdot 006)^8$, to three significant figures.

9. (a) Differentiate from first principles $3x^2$ with respect to x.
What is the slope of the tangent to the curve $y = 3x^2 - 2x + 1$ at the point where $x = -2$?

(b) Differentiate with respect to x:
(i) $(x^3 - 3)(x^2 - x - 4)$,

(ii) $\dfrac{4x + 1}{x^2 + x + 2}$.

10. (a) Evaluate

(i) $\displaystyle\int_3^5 3\,dx$ (ii) $\displaystyle\int_1^2 (1 + t)dt$ (iii) $\displaystyle\int_2^3 (1 + x)(1 - x)dx$.

(b) A stone is thrown downwards from the top of a high cliff. The distance, in metres, the stone has fallen after t seconds is given by

$$s = \int_0^t (3 + 10x)dx.$$

Find how far the stone has fallen after 2 seconds and show that the stone has fallen through a distance of 28 metres in the third second.