**AN ROINN OIDEACHAIS**

**LEAVING CERTIFICATE EXAMINATION, 1973**

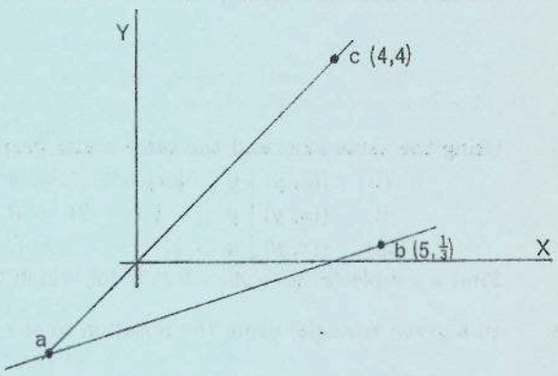
MATHEMATICS — ORDINARY LEVEL — PAPER I
(300 marks)

MONDAY, 11 JUNE—MORNING, 9.30 to 12.

**Six** questions to be answered.
All questions are of equal value.
Mathematics Tables may be obtained from the Superintendent.

1. A regular hexagon is inscribed in a given circle (radius $r$) and another regular hexagon is circumscribed about the circle. Find the ratio of the area of the inscribed hexagon to that of the circumscribed hexagon.

2. The equations of the lines forming two of the sides of the triangle $abc$ are $3y = x - 4$ and $y = x$ (see diagram).




Find

(i) the coordinates of $a$;

(ii) the length $|\,bc\,|$;

(iii) the equation of the line through $c$ perpendicular to $ab$;

(iv) the image of the triangle $abc$ by the axial symmetry (reflection) in the $x$-axis.

3A. The equation of a given circle is $x^2 + y^2 = 25$.

Show that the point (3, 4) belongs to the circle and find the equation of the tangent to the circle at the point (3, 4). Find also the area of the triangle formed by this tangent and the two axes.

**or**

3B. Show with proof how to divide a line segment internally into two parts so that the rectangle contained by the whole line segment and one part equals the square on the other part.

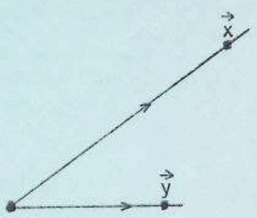
Assuming that the shorter side of the rectangle measures 10 cm, compute the area of the square.

4. Prove that the internal bisector of the vertical angle of a triangle divides the base in the ratio of the other two sides.

$abc$ is a triangle in which $|\,ac\,| = |\,bc\,| = 13$. $af$ is perpendicular to $bc$ such that $|\,af\,| = 12$ and $f$ is a point in $bc$. Compute the distance of the orthocentre $o$ from $a$.

[Note: The orthocentre is the point of intersection of the perpendiculars from the vertices to the opposite side.]

5. (*a*) In the diagram a vector $\vec{x}$ and a vector $\vec{y}$ are shown. Copy the diagram and then clearly show the vectors




(i) $\vec{x} + \vec{y}$,

(ii) $\vec{x} - 2\vec{y}$,

(iii) $\vec{y} + 2(\vec{x} - \vec{y})$.

(*b*) If $t$ is a real number such that $|\,t\,| = 2$ and if $\vec{v}$ is a non-zero vector, represent $\vec{tv}$

(i) if $t$ is positive,

(ii) if $t$ is negative.

Explain the equation $|\,\vec{tv}\,| = |\,t\,|\;|\,\vec{v}\,|$.

6. $S_a$ is a central symmetry with centre $a$.

(a) Justify each of the following statements and illustrate each by an example:

(i) $S^{-1}{}_a = S_a$

(ii) $S_a \circ S_a = 1_\pi$.

Prove that the composition of two axial symmetries (reflections) in perpendicular axes is a central symmetry.

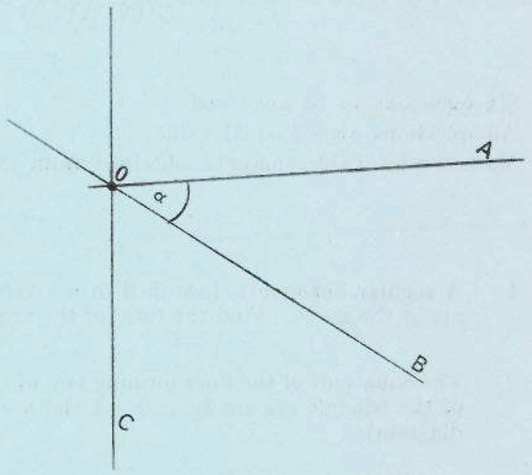
(b) Given $\{a, p, q\} \subset \Pi$ and $(p, q) \in S_a$, illustrate lines $L$ and $M$ not containing $p$ or $q$ such that $S_L \circ S_M = S_a$. Is $S_L \circ S_a = S_M$? Explain your answer.

7. $A$, $B$, $C$ are three lines concurrent at $o$ (see diagram)

If $(a, b)$ is a couple of the rotation $S_B \circ S_A$, show that $< aob$ measures $2\alpha$, and hence find a line $X$ through $o$ such that

$$S_B \circ S_A = S_C \circ S_X$$

Deduce that the composition of three axial symmetries in concurrent axes is an axial symmetry in an axis concurrent with these three. Hence, or otherwise, prove that the bisectors of the three sides of a triangle are concurrent.




8. Using the same axes and the same scales graph each of the following:

(i) $\{(x, y) \mid y \geqslant \frac{1}{2}x\} = A$

(ii) $\{(x, y) \mid y \leqslant -\frac{1}{2}x + 2\} = B$

(iii) $\{(x, y) \mid x \geqslant \frac{1}{2}\} = C$.

Find a couple $(x, y) \in A \cap B \cap C$ for which $3y - 2x$ is a minimum.

9. In a given triangle, using the notation in the Tables, page 9, prove that

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

The lengths of the sides of a triangle are in the ratio 4 : 5 : 6. Find, to the nearest degree, the measure of the greatest angle of the triangle.

10. Using the same axes and the same scales draw the graph of the function $f$: $f(x) = \sin x$, and the graph of the function $g$: $g(x) = 1 + \sin x$ for $-2\pi \leqslant x \leqslant 2\pi$ in each case.

From your graphs find the maximum and minimum values, range and period of each of the two functions.

For what values of $x$ in the given domain is $g(x) = g(-x)$?