AN ROINN OIDEACHAIS M.48

LEAVING CERTIFICATE EXAMINATION, 1972

MATHEMATICS - ORDINARY LEVEL - PAPER II (300 marks)

TUESDAY, 13th JUNE - Morning, 9.30 to 12

Six questions to be answered.
All questions are of equal value.
Mathematics Tables may be obtained from the Superintendent.

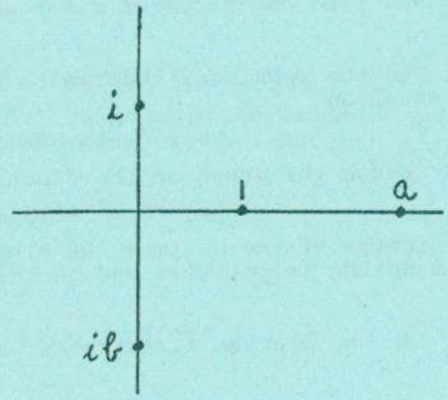
1. Calculate the compound interest on £900 for two years at 10% per annum.

A person borrowed £900 at a compound interest of 10% per annum. At the end of the first year he paid back £y. Find in terms of y (i) the interest for the second year, (ii) the amount he owed at the end of the second year. If another repayment of £y at the end of the second year would clear his debt, find the value of y to the nearest £1.

2. In an arithmetic series the first term is a and the common difference is d; show that the sum to n terms of the series is $\frac{n}{2}\{2a + (n - 1)\,d\}$.

The first term of an arithmetic series is -19 and the fifth term is -11. Find the least value of n for which the sum to n terms of the series (i) exceeds 0, (ii) exceeds 800.

3. (a) Make a rough copy in your answer book of the Argand diagram shown here $(a, b \in R)$, and on your diagram mark the points which represent the complex numbers (i) $a - ib$, (ii) $-a + ib$, (iii) $(a + ib) - (a - ib)$.



(b) Show that $3 + 4i$ is a root of the equation $x^2 - 6x + 25 = 0$ and find the other root.

Is $3 + 4i = 3 - 4i$? Give a reason.

Is $|3 + 4i| = |3 - 4i|$? Give a reason.

(c) If $z = x + iy$, plot on an Argand diagram four complex numbers z, such that $|z| = 5$.

4 A. Illustrate, by one example in each case, each of the following:

(i) A binary operation on a set such that the set is closed under the operation.

(ii) A binary operation on a set such that the set is not closed under the operation.

(iii) A non-binary operation on a set.

The set $\{a,b,c\}$ under the operation $*$, is a commutative group of order 3 (see table).

*	a	b	c
a	a	b	c
b	p	q	r
c	x	y	z

What is the identity element ?

If c is the inverse of b, write down the values of p, q, r, x, y, z in terms of a, b, c.

Show that $\left(\{0,1,2,3\}, + \pmod 4\right)$ is a group (you may assume associativity) and write out the Cayley table. Use your table to solve

$$3x + 1 = 3 \pmod 4.$$

(Note: $3x$ means $x + x + x$)

OR

4 B. The marks of 7 candidates in two tests are given in the table:

	Candidates Marks							Mean Mark	Standard Deviation
Test 1	45	50	55	60	65	70	75	$\bar{x}_1$	σ_1
Test 2	6	8	10	12	14	16	18	$\bar{x}_2$	σ_2

Calculate $\bar{x}_1$, $\bar{x}_2$, σ_1, σ_2 and show that

$$\frac{50 - \bar{x}_1}{\sigma_1} = \frac{8 - \bar{x}_2}{\sigma_2}.$$

Would you agree that the last candidate who scored 75 and 18 did relatively better in Test I than in Test 2 ? Give a reason for your answer.

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5. Explain by means of diagrams, or otherwise, each of the following:

relation, function, injection, surjection. Say, giving a reason, whether each of the following functions is an injection, a surjection or neither. Illustrate by a diagram in each case:

$$f : N \to N \quad : x \to x^2$$
$$g : R \to R \quad : x \to x^2 + 4x + 3$$
$$h : R^+ \to R \quad : x \to x^2 + 4x + 3$$
$$k : R \to R \quad : x \to 3x - 1$$

(Note: $R^+ = \{x | x \geq 0,\ x \varepsilon R\}$).

6. (a) The relations R and S are defined by

$$R = \{(a,c),\ (a,b),\ (b,a)\}; \quad S = \{(b,c),\ (c,b)\}.$$

Write down the domain and image of each relation.
Write down, also, the elements of

$$R^{-1},\ R \circ S,\ S \circ R$$

and show that $S^{-1} = S$.

(b) The functions f and g are defined by

$$f: R \to R : x \to \sin x \ (\text{ i.e. } f(x) = \sin x).$$
$$g: R \to R : x \to \pi x \ (\text{i.e. } g(x) = \pi x).$$

Write down the image (range) of f and the values of $f(0)$, $g(0)$, $f(\pi)$, $g(1\setminus\pi)$.
Investigate the truth or otherwise of the statement

$$f\{g(x)\} = g\{f(x)\}$$

and find one value of x for which $\pi f(x) = g(x)$.

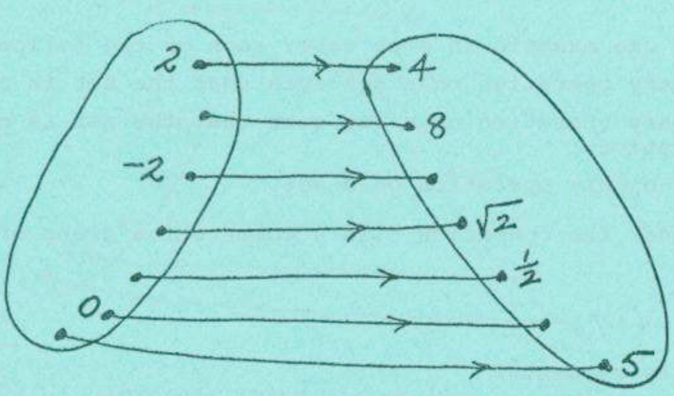
7. Use the remainder theorem to find the factors of $6x^3 - 19x^2 + 11x + 6$ and hence solve the equation

$$6x^3 - 19x^2 + 11x + 6 = 0.$$

Sketch the graph of the function

$x \to 6x^3 - 19x^2 + 11x + 6$, $(-1 \leq x \leq 3)$, and find from your graph the coordinates of the maximum and minimum turning points. Find also the domain of x for which the function is positive and increasing.

8. (a) The diagram illustrates part of the function $x \to 2^x$:



Copy the diagram into your answer book and fill in the missing numbers.

(b) Use the table.

x	·125	·25	·5	1	2	4	8
$\log_2 x$	-3	-2	-1	0	1	2	3

to graph the function $x \to \log_2 x$ in the domain $\cdot 125 \leq x \leq 8$ and use the graph to estimate the values of

$$\log_2 3,\ \log_2 27,\ \log_2 \tfrac{4}{5},\ \log_2 \sqrt{8}.$$

9. (a) Differentiate from first principles $2x^2 - 1$.
Draw a rough graph of the function

$f: x \to 2x^2 - 1 = f(x)$ and indicate on it the geometric meaning of $f'(1)$.

[Note: $\frac{df(x)}{dx} = f'(x)$]. Find the co-ordinates of the points on the graph at which

(i) $f'(x) = -1$, (ii) $f'(x) = 0$.

(b) Differentiate (i) $(x + 1)^2$, (ii) $\frac{x}{(x + 1)^2}$.

10. (a) Evaluate (i) $\int_0^2 2\, dx$; (ii) $\int_1^2 (1 + 2x)\, dx$. (iii) $\int_0^k (x^2 + x)\, dx$.

(b) The distance in feet, $S(t)$, travelled by a car in the first t seconds is given by

$S(t) = \int_0^t (1 + 2x)\, dx$. What distance does the car travel in the first 20 seconds ?

Verify that $S(20) - S(10) = 310$ and interpret this equation.