AN   ROINN   OIDEACHAIS

LEAVING   CERTIFICATE   EXAMINATION,   1972

MATHEMATICS - ORDINARY LEVEL - PAPER I

(300 marks)

THURSDAY, 8th JUNE - MORNING, 9.45 to 12.15

Six questions to be answered.
All questions are of equal value.
Mathematics Tables may be obtained from the Superintendent.

1. A rectangular oil-tank has the following internal dimensions: length $4\frac{1}{2}$ feet; width 3 feet; height $2\frac{1}{2}$ feet.   If a cubic foot of oil is equivalent to $6\frac{1}{4}$ gallons, give the capacity of the tank (i) in cubic feet, and (ii) in gallons.
   A second rectangular tank has twice the capacity of the first, and the ratios of its length, width, and height to each other are the same as the ratios of the length, width and height of the first tank to each other.   Compute the dimensions of the second tank in feet correct to three significant figures.

2. Write down the slope of the line which contains the two points (3, -1) and (-3, 2). Hence find the equation of the line and also

   (i) the length of the intercept which it cuts off on each of the axes,
   (ii) the perpendicular distance from the origin to the line.

3. A.   Is the point (2, -4) inside, on, or outside the circle $x^2 + y^2 = 24$?   Explain your answer.   Write down the length of the radius of the circle, and determine whether or not the line $x + y = 4\sqrt{3}$ is a tangent.

OR

3. B.   Show, with proof, how to construct an isosceles triangle having each of the angles at the base double the angle at the vertex.

4. (a) Prove that the areas of similar triangles are in the ratio of the squares of corresponding sides.

   (b) Three similar triangles are constructed on a line segment [xy] of length 5 inches.   (See diagram). The areas of the triangles are in geometric progression and the area of the greatest triangle is four times that of the smallest.   Show that the area of the smallest triangle is half that of the middle one and then compute the length of the base of each triangle.
   (You may use the sign $\sqrt{}$ in your answer.)
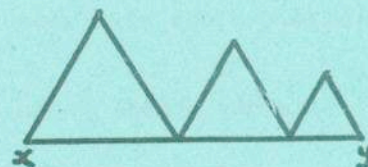
5. (a) Which of the following maps (i) a translation, (ii) a rotation, (iii) an axial symmetry (reflection) is always its own inverse ?   Give your reasons.

   (b) K is a circle which has c as centre.   D is a line containing c.   T is a tangent to K and $T \cap D = \{p\}$.   (See diagram).   Illustrate the image of K by the rotation $S_T \circ S_D$, and then write down the inverse of that rotation as a composition of reflections.
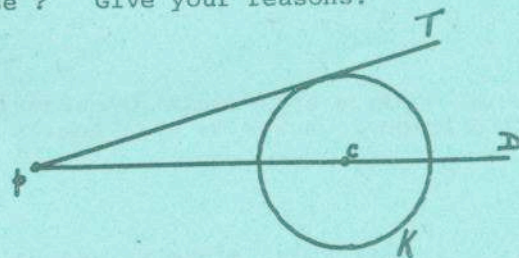
6. Give a definition of a translation of the plane Π, and draw a diagram to illustrate an example of a translation.
   Prove that the composition of two reflections (axial symmetries) in parallel axes which are disjoint is a translation.   Deduce that the composition of three reflections in parallel axes which are disjoint is again a reflection.

   $pqrs$ is a rectangle, and $A||B$ such that $|pt| = |ts|$. (See diagram).   Find the image of the rectangle by the composite reflection $S_C \circ S_B \circ S_A$.
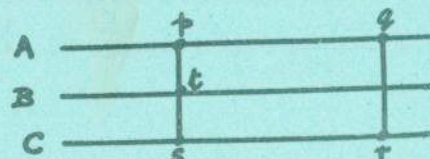
7. In the diagram o, x, y are three collinear points and o, w, z are also collinear. r and t are scalars such that $\overrightarrow{ox} = \vec{a}$ and $\overrightarrow{oy} = t.\vec{a}$; $\overrightarrow{ow} = \vec{b}$ and $\overrightarrow{oz} = r.\vec{b}$.

Find in terms of $\vec{a}$, $\vec{b}$ and r the vector $\overrightarrow{op}$ where p is the midpoint of $[xz]$.   If q is the midpoint of $[wy]$, find $\overrightarrow{oq}$ in terms of $\vec{a}$, $\vec{b}$ and t.

8. A small elevator is used to hoist concrete blocks to the top of a new building.   Each hour the elevator can lift a maximum load of 20 cwt.   The blocks are of two types, heavy blocks which weigh 4 stone each and light blocks which weigh 1 stone each.   If the work requires that <u>not</u> more than half the blocks hoisted are light, what is the greatest number of blocks which can be hoisted in an hour ?   (Note: 1 cwt. = 8 stone)

9. (a) Given that $\cos\beta = \frac{5}{6}$, compute the area of the triangle shown in the diagram.   (You may use the sign $\sqrt{}$ in your answer).
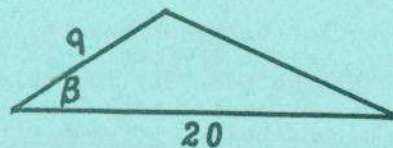
   (b) The angle of elevation of the top of a vertical factory chimney is observed to be 70° when measured from a point on the ground 100 feet from the chimney base. Assuming the ground is level what is the measure of the angle of elevation at a point on the ground 200 feet from the chimney base ?   (Give your answer to the nearest degree).

10. (a) Graph the function $f : x \to 1 - \cos 2x$, $0 \leqslant x \leqslant 2\pi$.   State the period and the range of the function.   Write down a periodic function which has period π and range $[-2, +2]$.

   (b) The diagram shows a periodic function of the form $g(x) = \alpha + \beta \sin x$, where α and β are integers.   Find α and β.