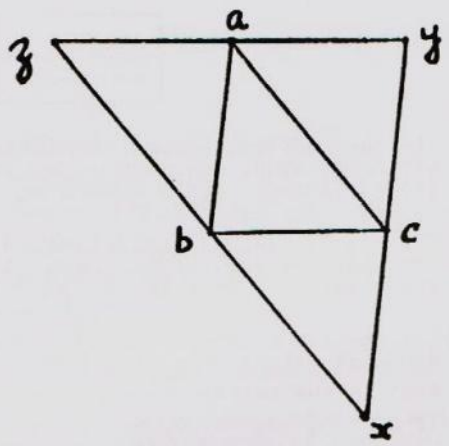
1971

PAPER I

1. A thin walled rectangular vat of length 6 metres and of breadth 5 metres contains liquid. Some of the liquid is poured into a cylindrical cask of internal diameter 70 cm. If the level of liquid in the vat falls by 0·85 cm., to what height will the liquid rise in the cask?
Had the diameter of the cask been twice as great, to what height would the liquid have risen?
Give your answer in each case correct to the nearest cm.

2. Prove that the perpendicular bisectors of the sides of a triangle are concurrent.
abc is any triangle. Through the vertices *a*, *b*, *c* lines are drawn which are parallel, respectively, to *bc*, *ca* and *ab*. (See diagram). Prove that the orthocentre of Δabc is the circumcentre of Δxyz.



3. (i) Find the distance, d, between the points (-2, -3) and (3, 9) and verify that the distance between the images of these points by reflection in the x-axis is also d.
(ii) *abc* is a triangle with vertices $a(1, 5)$, $b(3, 7)$ and c (5, 1). The Δpqr is the image of the Δabc by the translation $(1, 5) \rightarrow (0, 0)$. Find the coordinates of p, q, r. Hence, or otherwise, find the area of Δabc.
(iii) Find the equation of the line which contains the point $(\frac{1}{3}\ \ -2)$ and which is parallel to the line $y = 3(x + 1)$.

4. A. The circle $4x^2 + 4y^2 = 25$ is K and L is the line $6x + 8y = 25$. Verify that the point $(1\frac{1}{2}, 2)$ belongs to the intersection of K and L (i.e. $(1\frac{1}{2}, 2) \in K \cap L$) and also prove that L is a tangent to K.
Q is another line perpendicular to L and $(5\frac{1}{2}, -1) \in Q$.
Is Q a tangent to the circle? Give your reason.

OR

B. $[ab]$ is a line segment one unit in length and it is divided internally at x such that $|ab| . |bx| = |ax|^2$. If $|ax| = t$, show that $t = \frac{1}{2}(\sqrt{5} - 1)$ and deduce that $|ax| > |xb|$.
If $y \in [ax]$ such that $|ay| = |xb|$, prove that $|ax| . |xy| = |ay|^2$

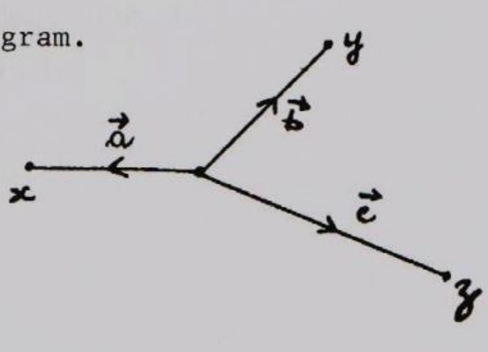
5. (i) Illustrate by a diagram that a parallel projection of the plane on a given line is a function. Is the inverse of the parallel projection also a function? Explain your answer.
(ii) $[ac]$ is a diagonal of a parallelogram $abcd$. Find the images of the points a, b, c, d by the projection on bd where the projection is parallel to ac.
Does parallel projection preserve lengths of line segments?
Does parallel projection preserve the ratios of the lengths of any two line segments?
Explain your answer in each case.
(iii) The central symmetries S_k , S_n, S_w, S_x are such that

$$S_k \circ S_n = S_w \circ S_x$$

Express S_x in terms of the other three. Find the image of a Δabc by the composition of central symmetries $S_c \circ S_b \circ S_a$.

6. (i) Show that the composition of two reflections in perpendicular axes is a central symmetry. Hence, or otherwise, deduce that the composition of two reflections in parallel axes is a translation.
(ii) A, B, C are three parallel lines. Find the line X such that $S_C \circ S_B \circ S_A = S_X$.
(iii) P, Q are parallel tangents to a circle K of centre k and R is a line such that $k \in R$ and $R || P$. Find the image of the circle K by $S_Q \circ S_R \circ S_P$.

7. $\vec{a}$, $\vec{b}$, $\vec{c}$ are three vectors as in diagram.
Construct
(i) $\vec{a} + \vec{b}$;
(ii) $\vec{b} - \vec{c}$;
(iii) $\vec{a} - \frac{1}{2}\vec{b} + \frac{1}{4}\vec{c}$.
Express each of the vectors
$\vec{xy}$, $\vec{yz}$, $\vec{zx}$ in terms of $\vec{a}$, $\vec{b}$ $\vec{c}$



and hence simplify the vector
$\overrightarrow{xy} + \overrightarrow{yz} + \overrightarrow{zx}$.
If $\vec{a} = -3\vec{i} + \vec{j}$, $\vec{b} = 3\vec{i} + 3\vec{j}$, $\vec{c} = 5\vec{i} - 3\vec{j}$.
Show that the Δxyz is right angled at y.

8. Using the same axes and the same scales indicate clearly each of the sets A, B, C and D in the co-ordinated plane where
   (i) $A = \{(x, y) \mid x \geqslant 0,\ y \geqslant 0,\ x + y \leqslant 4\}$;
   (ii) $B = \{(x, y) \mid y > x\}$; etc..
   (iii) $C = \{(x, y) \mid 0 \leqslant y \leqslant 2\}$;
   (iv) $D = A \cap B \cap C$.

   Find the co-ordinates of the point $(x, y) \in D$ for which $3x + 4y + 7$ has its minimum value.

9. (i) An athlete runs from a point p to a point q along a circular track of radius 60 metres and of centre c. If $\angle pcq$ measures 230^0, calculate the distance run to the nearest metre.
   (ii) If $\sin x = \frac{9}{41}$ $(0 \leqslant x \leqslant \pi)$, find, without using the tables, the value of cos x if $\sin x > \cos x$.
   (iii) Prove that $\tan^2 A - \sin^2 A = \tan^2 A.\ \sin^2 a$.

10. (i) The angles of a triangle are A, B, C and the sides opposite these angles are of length a, b, c respectively. Prove that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

   If $b = c = 6$cm and B has measure $75^c\ 32^1$, calculate a correct to the nearest cm.
   (ii) Write down the range (image) and the period of each of the following functions defined on R, the set of real numbers:

   (a) $x \to \sin x$;
   (b) $x \to 1 + \sin x$;
   (c) $x \to \sin 2x$;
   (d) $x \to \sin(x + 1)$.