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LEAVING CERTIFICATE EXAMINATION, 1970

MATHEMATICS (PASS) – PAPER II (300 marks)

MONDAY, 15th JUNE – Morning 9.30 to 12

Six questions to be answered.
All questions are of equal value.

N_0 is the set $\{1, 2, 3, 4, \ldots\}$
R is the set of real numbers
Mathematical Tables may be obtained from the Superintendent.

1. (a) Evaluate, correct to 2 places of decimals

$$\frac{3\cdot5 \times 10^{-3} \times (1\cdot21)^{\frac{1}{2}}}{1\cdot21 \times 10^{-4}}.$$

(b) Show that the inequality

$$(n+1)^{\frac{1}{n+1}} < n+1 \quad (n \in N_0).$$

implies the inequality

$$\left(1 - \frac{1}{n+1}\right) \log_{10}(n+1) > 0.$$

For what values of n is this latter inequality true ?
[Hint: $a.b > o$ implies a and b both have the same sign.]

2. (a) p and q are two numbers which can be written as non-terminating decimals with patterns as follows:

$$p = 2\cdot1011011101111011111011111 0 \ldots\ldots$$
$$q = 2\cdot2012011201112011112011111 2 \ldots\ldots$$

Say whether each of the numbers is rational or irrational.

(b) Give an example which shows that the sum of two irrational numbers can be a rational number.
Can the product of two irrational numbers be rational ? Explain your answer by giving an example.

(c) Prove that

$$\frac{(2+\sqrt{2}+\sqrt{3})(2+\sqrt{2}-\sqrt{3})}{6+8\sqrt{2}}$$

is a rational number.

3. (a) If A is any subset of U and if B is any subset of U where U is some non-empty set, say which of these statements are always true

$$(A \setminus B) \cap B = \phi; \quad B \setminus A = B \cap A'; \quad A \setminus B = B \setminus A.$$

[Note: A' is the complement of A in U].

(b) "A set which has n elements has 2^n subsets".
Verify this statement by writing down the subsets of a set

(i) which has 3 elements
(ii) which has 2 elements
(iii) which has 1 element
(iv) which is empty.

4. (a) The numbers $\frac{1}{n}$, $\sqrt{n}$ and $(-1)^n$ are the nth terms of the sequences $S^{(1)}$, $S^{(2)}$ and $S^{(3)}$ respectively. Which sequences have a limit ? Give very briefly the reasons for your answer in each case.

(b) A nursery man is developing a new strain of tulips. He finds that each tulip bulb sown will become 3 bulbs at the end of its growing season. How many years will it take to have available 10,000 bulbs assuming that there is only one growing season each year and that he begins with one bulb.

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5. (a) If $\log_{10} x = 2 \cdot 3010$ and $\log_{10} y = 1 \cdot 4771$ which of these are true

$\log_{10} (10y) = 14 \cdot 771$; $\log_{10} (\frac{x}{10}) = 1 \cdot 3010$; $\log_{10} (x + y) = 3 \cdot 7781$.

Write the correct version of each false statement.

(b) Show that the non-terminating binary number $0 \cdot \dot{0}\dot{1}$ $(= x)$, (i.e. $x = 0 \cdot 01010101 \ldots$) can be written in base ten as a geometric series in which the common ratio is $\frac{1}{4}$. Hence prove that $x = \frac{1}{3}$.

6. (a) If $x \in R$ and $x \neq 0$ differentiate $\frac{1}{x}$ with respect to x from first principles.

(b) Make a rough graph of the function f defined as $f(x) = \frac{1}{x}$; $x > 0$ [i.e. $y = \frac{1}{x}$]. Has the function a least value ? Give reasons for your answer. Calculate the rate of change of the function at the point $x = 3 \cdot 5$.

7. A factory employs skilled workers who earn £25 per week and unskilled workers who earn £15 per week. The total weekly wage bill must not exceed £800. At least 50 operators are required to work the machines, and at least 20 of these must be skilled. Factory regulations demand that at least half of all workers in the factory must be skilled.

If x is the number of skilled workers and if y is the number of unskilled workers write down the inequalities (other than $x \geqslant 0$, $y \geqslant 0$, $x + y \geqslant 0$) which govern x and y.

8. (a) (i) For what value of $\alpha \in R$ does the line with equation $y = \beta x + \alpha$ contain the origin ?

(ii) What point is always in the line $y = \beta x + 4$ for all $\beta \in R$?

(b) If the product of $y^2 + ky - 1$ and $2y + 3$ has no term in y^2 find the solution of the equation $y^2 + ky - 1 = 0$.

9. f and g are functions defined as follows:

$f(x) = \frac{x + 3}{x}$ provided $x \neq 0$ and $x \in R$

$g(x) = x(x + 3)$, $x \in R$.

Show that for every $x \neq 0$ both functions either have the same sign (positive or negative) or both functions are zero.

Solve the equation $g(x) = 4$.

10. (a) In how many ways can 2 things be selected from a set of 5 things ? How many subsets each containing r things are contained in a set of n things ?

(b) Three men enter a carriage in a train where there are 5 vacant seats. In how many ways can the men be seated ?

(c) How many functions can there be from a set of 5 elements to a set of 3 elements ? Explain your answer.