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LEAVING CERTIFICATE EXAMINATION, 1970

MATHEMATICS (PASS) – PAPER I (300 marks)

WEDNESDAY, 10th JUNE – Morning, 9.45 to 12.15

Six questions to be answered. All questions carry equal marks.
Mathematical Tables may be had from the Superintendent.
N is the set of natural numbers.

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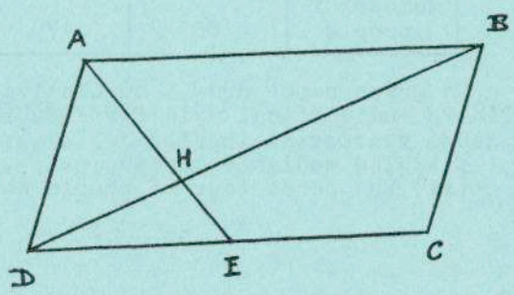
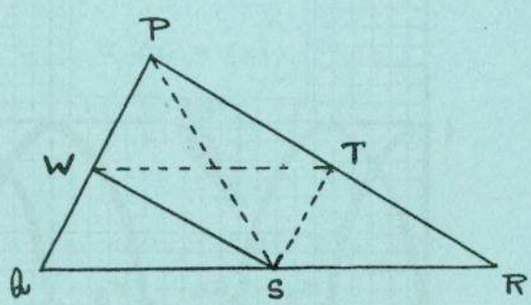
1. The base of a rectangular tank is horizontal and the tank is 150 centimetres long and 60 centimetres wide.
To provide central heating, oil is drawn off and burned at the rate of 2 litres per hour. If oil is being burned for 5 hours each day, find how far the level of oil in the tank will have fallen at the end of 30 days.
If the height of the tank is 120 centimetres, how many hours of burning will a full tank of oil provide ?
(1 litre = 10^3 cubic centimetres.)

2. (a) When is a number said to be rational ?
Write the real number $\frac{8}{7}$ as a decimal and show it is recurring.
If x is the recurring decimal $0{\cdot}\dot{1}2\dot{7} = 0{\cdot}127127\ldots$, show that $10^3x = 127 + x$ and hence, or otherwise, express x in the form $\frac{a}{b}$, where $a, b \in N$.
(b) Show how to divide a line segment in the ratio $\sqrt{2} : 1$.

3. Prove that if two triangles are equiangular, their corresponding sides are proportional.
ABCD is a parallelogram, as in diagram, and DE = EC. A line through E is drawn parallel to CB cutting BD in F and AB in G. Prove EF = $\frac{1}{2}$ EG.
By considering similar triangles, show that DH : HB = EH : HA, and deduce that DH : DB = 1 : 3.



4. If the vertical angle of a triangle is bisected internally, prove that the bisector divides the base of the triangle in the same ratio as the other two sides of the triangle.
State (without proof) the converse of the theorem.
PQR is a triangle and QS = SR. ∠PSR is bisected by ST and TW || RQ.
Prove that SW bisects ∠PSQ.



5. (a) Express (i) 108° in radians
(ii) $\frac{7\pi}{4}$ radians in degrees.
Use your tables to write down the value of $\sin\left(-\frac{11\pi}{4}\right)$ and of $\cos 200°$.
(b) If A and B are acute angles such that $\sin A = \frac{24}{25}$ and $\cos B = \frac{15}{17}$, find, without the use of tables, the value of $\sin(A + B)$.
If A, B, C are the three angles of a triangle, show, without using the tables, that the angle C is less than 90°.

6. (a) Prove that (i) $\sin^2 x + \cos^2 x = 1$
(ii) $\frac{\tan^2 \alpha}{1 + \tan^2 \alpha} = 1 - \cos^2 \alpha$.
(b) ABCD is a parallelogram and the diagonals intersect at K.
AK = 3 cm., KD = 7 cm., ∠AKD = 40°.
Calculate the length of each side of the parallelogram.
Give your answer correct to one place of decimals in each case.

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7. (i) The coordinates of A and B are (−2, 5) and (3, −7). Find the coordinates of K, the midpoint of AB.
Find the length of AB and of AK and verify that AB = 2AK.
M(2, 2) is the midpoint of the line segment PQ. If the coordinates of P are $(\frac{1}{3}, -\frac{1}{2})$, find the coordinates of Q.

(ii) The coordinates of two points L and M are (0, 0) and (4, 4), respectively. If N is a point below the line LM such that the area of △ LMN is constant and equal to 8, find the equation of the locus of N.

8. (i) Find the equation of the line through the point (−4, 2) which is perpendicular to the line $3x - 4y - 5 = 0$.

(ii) Lines of gradient $\frac{1}{3}$ and (−2) from a parallelogram and a diagonal of the parallelogram joins the vertices (2, 1) and (4, 4).
Find the coordinates of the other vertices of the parallelogram.

9. In a recent contest people were asked to estimate the number of beans in a bag. The following is a grouped frequency distribution of the estimates:

Estimate	201 to 250	251 to 300	301 to 350	351 to 400	401 to 450	451 to 500
Number of people	80	90	120	140	100	70

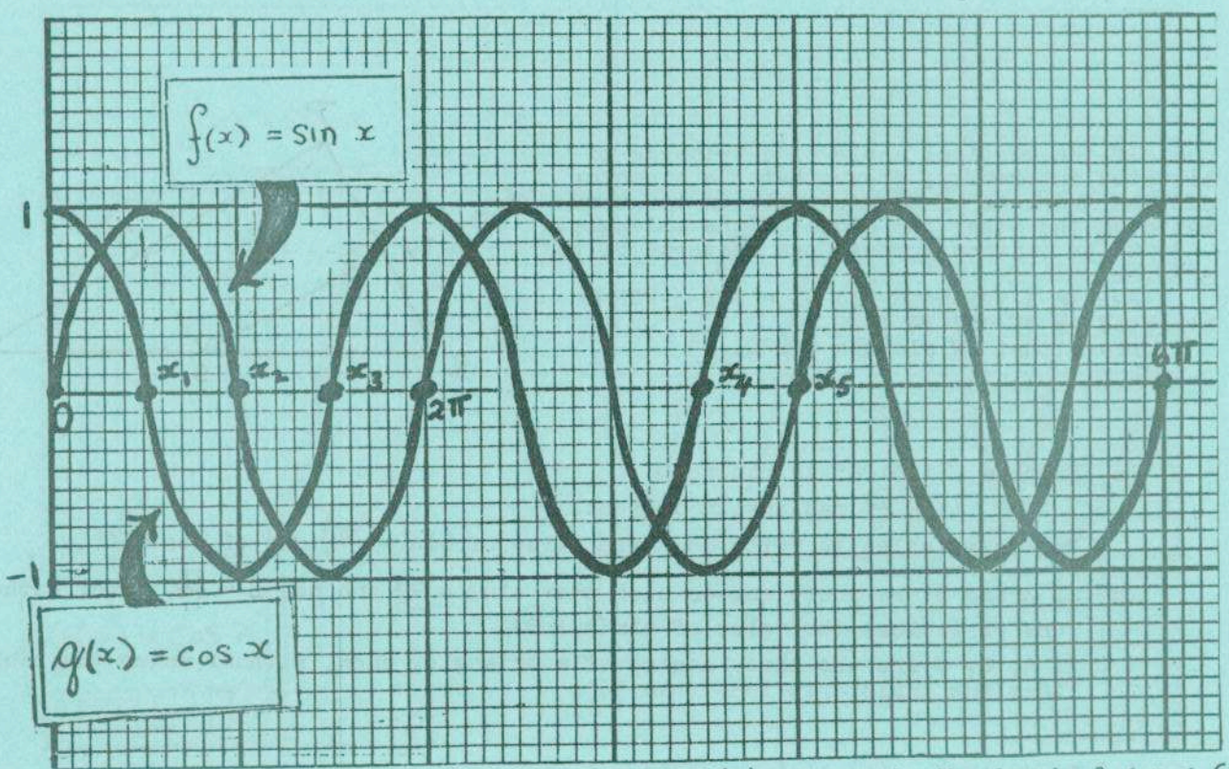
Which is the modal class (class of greatest frequency) of this distribution and how many people took part in the contest ?

Use the above distribution to complete the following cumulative frequency table:

Estimate	Less than 250	Less than 300	Less than 350	Less than 400	Less than 450	Less than 500
Number of people	80	170				

On graph paper draw a cumulative frequency graph (using horizontal axis for the ESTIMATE and vertical axis for FREQUENCY).
Use your graph to find
(i) The median estimate.
(ii) The percentage of people who estimated between 290 and 409 beans.

10.



The diagram shows the graph of $f(x) = \sin x$ and $g(x) = \cos x$ in the domain $0 \leq x \leq 6\pi$.
Write down the values of
(i) x_1, x_2, x_3, x_4, x_5, in terms of π,
(ii) $f(x_1)$, $g(x_1)$.
If $h(x) = f(x) + g(x) = \sin x + \cos x$, write down the values of $h(0)$, $h(x_2)$, $h(x_3)$, $h(2\pi)$.
Use the graph to find the domain of x for which
$\sin x \geq \cos x$, $0 \leq x \leq 6\pi$.