

Six questions to be answered.  
All questions are of equal value.  
Mathematical Tables may be obtained from the Superintendent.

N is the set of natural numbers  
Q is the set of rational numbers  
Z is the set of integers.

1. Show that  $\sqrt[3]{18}$  lies between 2.6 and 2.7.  
Hence, or otherwise, show that 
$$\frac{(3.98)^2 \times 13}{\sqrt[3]{18,005}} < 8.$$
2. (a) Plot on the number line the set  $\{x \mid x > 4x - 9, x \in Z \text{ and } x > 0\}$ .  
(b) If  $a, b \in Q$  and  $a < b$ , show that  
(i)  $\frac{a+b}{2} < b$ , (ii)  $a < \frac{a+b}{2} < b$ .  
If  $x, y \in Q$  and  $x < y$ , is it correct to say that  $\frac{x+y}{2}$  is the next greatest number after  $x$ ?  
Give your reason.
3. (a) Say about each of the following whether it is true or false and illustrate your answer by an example in each case given that A, B, X are sets:  
(i)  $A \cup A = A$ .  
(ii)  $B \cap B = \phi$ .  
(iii)  $A \subset B \implies A \cup B = A$ .  
(iv)  $A \cap B = B \implies A = B$ .  
(v)  $A \cap B = X \implies A \cap X = B$ .  
(b) If C is the set of cars in Ireland B the set of black cars in Ireland and I the set of cars assembled in Ireland, indicate on a diagram the set of black cars assembled in Ireland.  
What are the elements of  
(i) The set  $C \setminus (B \cup I)$ , (ii) the set  $(B \cup I) \setminus C$ ?
4. (a)  $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$  are the first three terms of a sequence. Is the sequence arithmetic? Is it geometric? Give your reasons. Write down the  $n$ th term, assuming the indicated pattern is followed.  
(b) Find the sum of the first 15 terms of the arithmetic series  $1 + \frac{1}{2} + 0 + \dots$ .  
(c) Show that  $S_n$ , the sum of the first  $n$  terms of the geometric series  $1 + 2 + 4 + \dots$  is  $2^n - 1$ . If  $T_n$  is the  $n$ th term of this series, show that  $T_n = 1 + S_{n-1}$ .
5. (a) (i) Transform 57.25 from base 10 to base 2.  
(ii) Divide the binary number 1,110,010 by the binary number 1,110.01 and express your answer in binary notation.  
(b) Write down the first three terms of the binomial expansion  $(3+x)^4$  and use this expansion to evaluate  $(3.03)^4$  correct to four places of decimals.
6. The line  $y = mx + 1$  is perpendicular to the line  $x + 2y = 3$ ;  
(i) find the value of  $m$ ,  
(ii) show that the line  $y = (m-1)x$  is perpendicular to the line  $x + y = 0$ .  
Graph the set of points satisfying the simultaneous conditions  
(a)  $x + y \leq 0$ , (b)  $x \geq 0$ , (c)  $y \geq x - 2$ .
7. Differentiate  $y = x(1-x)$  with respect to  $x$  from first principles.  
Find the slope of the tangent to the curve  $y = x(1-x)$  at the point  $(\frac{1}{2}, \frac{1}{4})$  and hence or otherwise find the equation of the tangent at that point.  
Draw a rough sketch of the curve  $y = x(1-x)$ .
8. (i) Show that  $\log_x \frac{M}{N} = \log_x M - \log_x N$ .  
(ii)  $x$  is a real number. For what values of  $x$  has  $\log_{10} x$  a meaning? Draw a rough graph of  $\log_{10} x$ .  
(iii) If  $\log_{10} \left( \frac{a}{a-1} \right) = 3$  and  $\log_{10} (a^2 - 1) = 2$ , find the value of  $a(a+1)$ .  
(iv) Solve the simultaneous equations:  
$$p^3 q = 3^{16}$$
$$\log_3 p + \log_3 q = 5.$$
9. (a) There are 5 seats in a car. In how many ways can 3 men seat themselves to travel in the car if only two of them are able to drive?  
(b) In a café a menu card lists the choices for a 4-course lunch. The first course is soup or else fruit juice. The second course is one of 3 meat-dishes. The third course is any one of 3 deserts. The final course is tea or coffee. How many different lunches can the café provide?
10. Is the relation "less than" on the set R of real numbers a function? Explain your answer.  $f, g$  and  $h$  are three functions whose domain is the set  $A = \{3, 1, 2\}$ .  $g(x) = \frac{1}{f(x)}$  and  $h(x) = f(x) + 1$  for all  $x \in A$ .  
(i) If the range of  $f$  is  $\{4, 2, 3\}$ , write down the range of  $g$  and the range of  $h$ .  
(ii) If  $f(3) = 3$  and  $f(1) = 4$ , find  $g(2)$  and  $h(1)$ .