LEAVING CERTIFICATE EXAMINATION, 1968

MATHEMATICS (PASS) – PAPER II (300 marks)

MONDAY, 17th JUNE – Morning 10 to 12.30

Six questions to be answered.
All questions are of equal value.
Mathematical Tables may be obtained from the Superintendent.

1. (a) Show that the value of $(11 \cdot 09)^2 \; (2 \cdot 01)^{\frac{5}{2}} \; (0 \cdot 0143)^{-\frac{1}{2}}$ exceeds $5 \cdot 5 \times 10^3$.

   (b) If x and y are positive integers, deduce which of them is the greater in each of the following cases:

   (i) $\frac{1}{x} > \frac{1}{y}$,

   (ii) $-y < -x$,

   (iii) $x - y > y - x$,

   (iv) $\frac{x - y}{y} > \frac{y - x}{x}$.

2. (a) x is an even integer. Write down the elements of the set $\{x | 0 < x + 2 \leqslant 17 - 2x\}$.

   (b) Let the universal set $U = \{x | 3 \leqslant x < 7\}$ where x is a real number.
   Let $A = \{x | 3 < x < 5\}$, $B = \{x | 4 < x < 6\}$.

   What is (i) $A \cup B$, (ii) $A \cap B$, (iii) A' ?
   [Note: A' is the complement of A].

3. A child has 40 wooden blocks which he describes as being shaped either long or flat or cubic, no block having more than one of these shapes. Each block is either all red or all blue.
Twenty blocks are flat. Flat blocks are not red.
Thirty blocks are blue. Long blocks are not blue.
Red blocks are not cubic.
Find the number and colour of blocks which are (i) long, (ii) flat, (iii) cubic.

4. Write down an arithmetic progression whose first term is 3 and give the formula for its nth term.
The nth term of an arithmetic progression is $2n - 1$. Find the first term, the common difference and the sum of the first 20 terms (S_{20}).
Show also that $S_1 = 1^2$, $S_2 = 2^2$, $S_3 = 3^2$ and prove by induction that S_n (the sum of the first n terms) is equal to n^2.

5. (a) Convert the binary number 1100101 to denary.
   Write in binary form (i) $2\frac{1}{2}$, (ii) $1\frac{3}{4}$, (iii) $\frac{1}{8}$, (iv) $\frac{1}{3}$.

   (b) If $E = (1 + h)^5$ and $h = 0 \cdot 0012$ use the binomial expansion to evaluate E to three places of decimals.
   Show that the limit as h tends to zero of $\frac{E - 1}{h}$ is 5.

6. Graph the lines $x - 2y = 2$ and $3x = 2y$.
Solve the simultaneous equations

$$x - 2y = 2$$
$$2y - 3x = 0$$

and explain the solution by referring to your graph.
On your graph find the set whose elements simultaneously satisfy the inequalities

$$3x \geqslant 2y; \quad y \leqslant 0; \quad x - 2y \leqslant 2.$$

7. Sketch the graph of $y = 3 - x^2$. Estimate roughly the slope of the tangent at the the point (1, 2).
Differentiate $3 - x^2$ with respect to x from first principles and hence or otherwise find the equation of the tangent to the curve $y = 3 - x^2$ at the point (1, 2).

8. (a) Show that $\log_a MN = \log_a M + \log_a N$.

   (b) Show that $\log_x\left(\frac{a - b}{b - c}\right) + \log_x\left(\frac{c - b}{b - a}\right) = 0$.

   (c) If a_1, a_2, a_3 are in geometric progression, show that $\log_x a_1$, $\log_x a_2$, $\log_x a_3$ are in arithmetic progression.

9. A man purchases two types of coal, one costing £12 per ton and the other costing £10 10s. per ton. He mixes together the same weight of each type and sells the mixture at £13 15s. per ton. Find his percentage profit.
In what proportion by weight should he mix the two types of coal so that by selling the mixture at £13 15s. per ton his profit would be 25% ?

10. (a) A girl possesses three cardigans, two blouses and four skirts. From these she selects an outfit consisting of one cardigan, one blouse and one skirt. How many different outfits may she select ?

    (b) Each of four boys x, y, z, w has a solution to a mathematics problem. Each boy passes his solution to one of the other three. If x gives his solution to z we shall write this information as (x,z). The set of couples (ordered pairs) $P = \{(x,z), (z,y), (y,w), (w,x)\}$ explains the interchange.

    (i) Is P a function ?
    (ii) If $S = \{x,y,z,w\}$ is $P \subset S \times S$?
    (iii) Give another subset of $S \times S$ which is a function having S as both domain and range.