LEAVING CERTIFICATE EXAMINATION, 1968

MATHEMATICS (PASS) - PAPER I (300 marks)

WEDNESDAY, 12th JUNE - Morning, 10 to 12.30

<u>Six</u> questions to be answered. All questions carry equal marks.
Mathematical Tables may be had from the Superintendent.

1. A solid spherical iron ball weighs 800 grams. If 1 cubic centimetre of iron weighs $7\frac{1}{2}$ grams calculate the radius of the ball to the nearest millimetre.
If the sphere were converted into a right circular cone having the diameter of its base equal to the diameter of the sphere what would be the height of the cone to the nearest centimetre ?
[Take $\pi = \frac{22}{7}$] .

2. Let S be the geometric series $a + ar + ...$
Show that the sum to n terms (S_n) of the series is $\frac{a(r^n - 1)}{r - 1}$.
Show that the odd terms of the series form another geometric series whose sum to n terms is $\left(\frac{r^n + 1}{r + 1}\right)S_n$.

3. If two triangles have an angle of the one equal to an angle of the other, and the sides about the equal angles proportional, prove that the triangles are similar.
ABC is a triangle. X is the midpoint of AB and Y is the midpoint of AC. Prove that XY is parallel to BC and that $XY = \frac{1}{2}$ BC.

4. Prove $\cos(A + B) = \cos A\cos B - \sin A\sin B$. [You may confine your argument to the case where $A + B < \frac{\pi}{2}$].
Hence deduce (i) $\cos 2\theta = 1 - 2\sin^2\theta$,
(ii) $2\cos^2\frac{\pi}{16} = 1 + \cos\frac{\pi}{8}$.

5. Prove that the areas of similar triangles are proportional to the squares on their corresponding sides.
A triangular field is divided in two halves by a fence which runs parallel to one side. If the length of the perimeter of the field is 280 yards calculate the length of the perimeter of the triangular plot cut off by the fence. [Take $\sqrt{2} = 1\cdot4$].

6. (a) Two of the following four functions are periodic and the domain of each function is $\{x | x \geq 1,\ x \in R\}$:
$f(x) = \frac{1}{2}\sin x$; $f(x) = \cos x - 1$; $f(x) = \frac{1}{x}\sin x$; $f(x) = x - \cos x$.
Determine the period of each of the periodic functions.
(b) The set of ordered pairs $\{(x, f(x)) | f(x) = \sin\pi x\}$ is a function. The domain is $-1 \leq x \leq +1$. Find the range and sketch the graph of the function.
Find values for x_1 and x_2 in the given domain such that $x_1 < x_2$ but $f(x_1) > f(x_2)$.

7. (i) Use your tables to find the following:
$\sin 120°$, $\cos(-20°)$, $\sin\frac{\pi}{4}$.
(ii) In a triangle ABC prove that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.
(iii) The sides AB and AC of a triangle ABC are equal in length. AB is 4 feet in length. $\angle ABC = 45°$. Calculate the length of BC correct to the nearest inch.

8. Find the equation of the line containing the points A(1,4) and B(3,-2).
Also (i) show that the line contains the point (-3,16);
(ii) find the slope of the perpendicular to the line;
(iii) find the equation of the line which contains the origin and which is parallel to AB.

9. (i) By combining the areas of two triangles, or otherwise, find the area of the parallelogram whose vertices are A(0,0), B(2,1), C(3,3), D(1,2).
(ii) A(0,0), X(-1,2), Y(-2,-1) and Z are points, and AXYZ is a parallelogram. Find the coordinates of Z.

10. The number of peas in each pod in a sample of 100 pods is expressed in the following table:

Number of peas in pod	0	1	2	3	4	5	6	7	8
Number of pods	2	6	11	16	25	18	10	8	4

Calculate the average number of peas per pod.
Find the mean deviation of the distribution.
Draw on graph paper a histogram to represent the distribution.
Could more than half the peas be contained in a selection of 40% of the pods ?
Explain your answer by reference to the table.