M.42

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LEAVING CERTIFICATE EXAMINATION, 1967

MATHEMATICS (PASS) - PAPER II (300 marks)

TUESDAY, 13th JUNE - Morning 10 to 12.30

Six questions to be answered.
All questions are of equal value.
Mathematical Tables may be obtained from the Superintendent.

1. (a) By making approximations, or otherwise, show that the value of

$$\frac{(1 \cdot 1276 \times 2 \cdot 0813)^3}{\sqrt{0 \cdot 000381} \times (4 \cdot 938)^2}$$

is greater than 16.

(b) Find, correct to two significant figures, the value of

$$(17 \cdot 05)^2 \times (0 \cdot 0092)^3 \times \sqrt[5]{235 \cdot 8}.$$

2. (a) (i) Find the values of x for which

$$11 < 2x - 1 \leqslant 23 - x, \ x \text{ integral}.$$

(ii) Write down the domain of values of x for which

$$11 < 2x - 1 \leqslant 23 - x, \ x \text{ real}.$$

(b) When is a natural number said to be prime ? Write down all the prime numbers between 1 and 16.

Let $U = \{x \mid x$ is a natural number and $1 \leqslant x \leqslant 16\}$. Let A, B, C be subsets of U such that A is the set of squares in U (i.e. 1, 4, 9, 16), B the set of even numbers in U and C the set of prime numbers in U. What are the sets

(i) $A \cap B$, (ii) $(A \cup B \cup C)'$, (iii) $A \cap C$?

Draw a diagram to represent the sets U, A, B, C and show all the elements of the sets.

3. In a class of 35 students, 23 study French, 20 study Drawing, and 5 students study Botany but not French or Drawing. 5 students study both French and Botany, 8 students study both Botany and Drawing and 15 students study both Drawing and French. If only 5 students of the class study all three subjects, how many students of the class do not study any one of the three subjects ?

4. (i) Show that the sequence whose nth term is $2 + 3n$ is an arithmetic progression.

The nth term of an arithmetic progression is $a + bn$ (a, b independent of n). Find a and b if the 4th term is $2\frac{1}{4}$ and the 10th term is $5\frac{1}{4}$. Find also the sum of the first 20 terms.

(ii) Find a value of n for which $\frac{3}{n}$ is less than $0 \cdot 01$ (n natural).

Write down the limit of $\frac{3}{n}$ as n tends to infinity.

Deduce the limit of $\frac{2n + 3}{n}$ as n tends to infinity.

5. (a) (i) Transform $110010 \cdot 101$ from base 2 to base 10.

(ii) Express the product of the two binary numbers 1010 and $1 \cdot 11$ in binary form.

(b) Write down the first four terms in the expansion of $(1 - 2x)^8$ and use this or any other expansion to find the value of $(0 \cdot 998)^8$ correct to four places of decimals.

6. Using the same axes and the same scales draw the graphs of $x + y = 1$ and $x - y = 1$ for values of x from $x = -1$ to $x = +3$.

On your graph shade in the set of ordered pairs (x, y) which simultaneously satisfy the three inequalities:-

$$x + y - 1 > 0, \quad x - y - 1 > 0, \quad 2 \geqslant x.$$

7. A window cleaner cleans all the windows of an office block in 4 hours, each window taking the same length of time to clean. If the time taken to clean each window could be reduced by 1 minute, the window cleaner could then clean half the number of windows of the office together with 6 more of its windows in 2 hours. Find the number of windows in the office.

8. (a) Prove that $\log_a MN = \log_a M + \log_a N$.

(b) If $\log_a\left(\frac{t}{1 - t}\right) = x$, $\log_a(1 - t^2) = y$ and $\log_a 10 = z$, express $\log_{10} t(1 + t)$ in terms of x, y, z.

(c) Solve the simultaneous equations:-

$$x^3 y^2 = 2^{12}$$

$$\log_2 x + \log_2 y = 5.$$

9. (a) Differentiate $x^2 + 4$ with respect to x from first principles.

Find the co-ordinates of the point in the curve $y = x^2 - 2x + 2$ at which the tangent is parallel to the x-axis.

(b) Solve the equation $2x^2 - 4x + 4 = 2x - 1$ and hence deduce that the curves $y = x^2 - 2x + 2$ and $2y = 2x - 1$ do not cut one another.

OR

9. (i) By evaluation or otherwise show that

$${}^7C_4 = {}^7C_3.$$

If ${}^nC_8 = {}^nC_5$, what is the value n ?

(${}^nC_r$ denotes the number of combinations of n things taking r at a time.)

18 boys in a class play hurling. In how many different ways can a team of 15 boys be picked for a hurling team ?

If one particular boy must always be included, how many different ways is it then possible to choose 15 ?

(ii) Prove by induction, or otherwise, that

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots + \frac{1}{(2n - 1)(2n + 1)} = \frac{n}{2n + 1}$$

for every $n \geqslant 1$ and n a natural number.

10. Define "Relation" and "Function".

Give an example of

(i) a relation which is a function,

(ii) a relation which is not a function.

Let $A = \{2, 3, 5\}$ and $B = \{4, 7\}$.

Write out and graph the set-product (Cartesian-product) $A \times B$ and show that $A \times B \neq B \times A$.

If $R = \{(x, y) \mid x \in A, y \in B$ and $y \geqslant 2x + 1\}$, write down and graph the elements of R and say whether R is a function or not a function. What is the domain and what is the range of R ?