AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1967

MATHEMATICS (PASS) - PAPER I (300 marks)

FRIDAY, 9th JUNE - Morning, 10 to 12.30

Six questions to be answered. All questions carry equal marks.

Mathematical Tables may be had from the Superintendent.

1. (i) A rectangular field is 194 yards long and 167 yards wide. Find the area of the field in acres, correct to two significant figures. [1 acre = 4840 square yards].

(ii) A cylindrical vessel of internal diameter 3 cm. contains water, the level of the water being 2 cm. from the top of the vessel. How many solid spheres of diameter 6 mm. must be dropped into the vessel to bring the level of the water to the top of the vessel, assuming that all the spheres are totally immersed in the water ?

2. (i) Is $3(2^n)$ equal to 6^n for all values of n ? Give the reason for your answer.

(ii) Find T_n, the nth term of the geometric progression 5, 15, 45, ... and find S_n, the sum of the first n terms. Show that for that progression $3T_n - 2S_n$ has the same value for all values of n.

(iii) The population of a certain country is now 10 million. If the population increases by 2 per cent from one year to the next, find, correct to the nearest million, what the population will be in 20 years' time.

3. (i) Given a line segment of unit length, show how to construct line segments of length $\sqrt{2}$ units and $\sqrt{3}$ units, respectively. Then show the points on a number line that correspond to the numbers 0, 1, 2, 3, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{3} + \sqrt{2}$, $\sqrt{3} - \sqrt{2}$.

(ii) Given two line segments, of length 1 unit and a units, respectively, show how to construct a line segment of length a^2 units.

4. If two triangles have an angle of the one equal to an angle of the other and the sides about the equal angles are proportional, prove that the triangles are similar.
Two straight lines AB, CD intersect at O. If AO.OD = CO.OB, prove that AC is parallel to DB.

5. (i) Using the Tables, find the values of the following:-

$\sin 50°$,	$\sin 130°$,	$\sin 230°$,
$\cos 23°14'$,	$\cos 156°46'$,	$\cos 383°14'$,
$\tan 53°22'$,	$\tan 126°38'$,	$\tan 413°22'$.

(ii) If a and b are two numbers such that $a > b$, does it follow that $\sin a° > \sin b°$? Give the reason for your answer.

(iii) Prove $\dfrac{\cos\alpha}{\cos\alpha - \sin\alpha} - \dfrac{\sin\alpha}{\cos\alpha + \sin\alpha} = \sec 2\alpha$.

6. What is the range of values of $\cos x$, the domain of x being the real numbers ?
Why is $\cos x$ said to be a *periodic* function ?
Draw a graph of $\cos x$ (where x is the radian measure of the angle) for values of x from 0 to 3π.
Find from your graph the values of x between 0 and 3π for which $\cos x = 0{\cdot}5$.

7. (i) A and B are two points on the circumference of a circle of centre O and radius 5 cm. The distance from A to B along the arc is 2 cm. Express the acute angle AOB (a) in radians, (b) in degrees, correct to the nearest degree. [For (b) see Tables p.30].

(ii) ABC is a triangle. AB = 5 cm., $\angle ACB = 30°$, $\angle BAC = 53°8'$. Find the length of BC. If BC is produced to D so that CD = 2 cm., find the length of AD in cm., correct to one place of decimals.

8. (i) The co-ordinates of two points A, B are (-1, 1) and (9, 5), respectively. Find the co-ordinates of M, the mid-point of the line segment AB. If M is also the mid-point of ON, where O is the origin, find the co-ordinates of the point N.

(ii) If (x_1, y_1), (x_2, y_2) are the co-ordinates of the points P, Q, respectively and O is the origin, show that the area of the triangle OPQ is $\left|\frac{1}{2}(x_1y_2 - x_2y_1)\right|$, i.e. the value of $\frac{1}{2}(x_1y_2 - x_2y_1)$ without regard to sign.

Find the area of the triangle whose vertices are the points (0,0), (5, 1), (1, 4). Find also the area of the triangle whose vertices are the points (3, 3), (8, 4), (4, 7).

9. R and S are two points whose co-ordinates are (-3, 1) and (3, 9) respectively. Find (i) the distance between R and S, (ii) the slope of the straight line through R and S, (iii) the equation of the straight line through the origin perpendicular to RS, and (iv) the co-ordinates of the point where the perpendicular from the origin cuts RS.

10. (a) Calculate either the mean deviation or the standard deviation of the five values 9, 10, 12, 13, 17.

(b) The following table shows the number of farms of various sizes in a certain district:

Size of Farm* (in acres)	0 - 10	10 - 20	20 - 30	30 - 50	50 - 100
Number of Farms	50	120	250	200	180

*Note: '0-10' means less than 10, '10-20' means at least 10 but less than 20, etc.

Draw a histogram to represent the distribution.
Calculate the arithmetic mean (on the assumption that the values are concentrated at the mid-points of the class-intervals).