M.42

A N   R O I N N   O I D E A C H A I S

LEAVING CERTIFICATE EXAMINATION, 1966

MATHEMATICS (PASS) – PAPER II (300 marks)

MONDAY, 13th JUNE – Morning 10 to 12.30

Six questions to be answered.
All questions are of equal value.
Mathematical Tables may be obtained from the Superintendent.

1.(a) By making approximations, or otherwise, show that the value of

$$\frac{(0\cdot18)^2(63\cdot89)^{\frac{1}{3}}}{\sqrt{0\cdot00263}}$$

is less than 10.

(b) Find, correct to two significant figures, the value of:-

$$\frac{4\cdot023 \times (2\cdot381)^3}{\sqrt{0\cdot7028}}.$$

2.(a)(i) Write down the values of x for which

$-2 < 2x + 1 \leqslant 4$, x integral.

(ii) Write down the domain of values of x for which

$-2 < 2x + 1 \leqslant 4$, x real.

(b) Let $A = \{-1, 0, 1\}$. When any element of A is multiplied by any element of A (this includes the multiplication of an element by itself), show that the product is an element of A.

When any element of A is added to any element of A, show that the sum is not necessarily an element of A.

3.(a) Let $U = \{1,2,3,4,5,6,7,8,9\}$ be the universal set and let $A = \{2,4,6,8\}$, $B = \{1,3,5,7\}$, $C = \{1,2,3,4,5\}$. Write down the elements of

(i) $A \cup C$, (ii) $B \cap C$, (iii) B', (iv) $(A \cup B)'$.

[(iii) and (iv) denote complement of B and complement of $(A \cup B)$, respectively.]

(b) Of the farmers in a certain district 50 sowed beet and 39 sowed kale and 20 of these sowed both beet and kale. If there were only 3 farmers who sowed neither beet nor kale, how many farmers were in the district ?

4.(i) The nth term of a sequence is $\frac{2n}{n+1}$. Write down the first four terms of the sequence and say which term of the sequence is $\frac{13}{7}$.

Is that sequence an arithmetic progression ? State your reason.

(ii) Prove by induction, or otherwise, that

$$1 + 3 + 5 + \cdots + (2n - 3) = (n - 1)^2.$$

OR

4. (i) Write down the nth term of the arithmetic series

$$19 + 16 + 13 + \cdots$$

and find the sum of the first 20 terms.

(ii) If n is a natural number, find for what values of n is $1 + \frac{2}{n}$ less than $1\cdot01$.

Write down the limit of $1 + \frac{2}{n}$ as n tends to infinity.

5.(a) Transform 13.5 from base 10 to base 2.

(b) Express the sum of the two binary numbers 1001 and 1101 in binary form.

(c) Show that $(3 + 2i) + (3 - 2i)$ and $(3 + 2i)(3 - 2i)$ are real, and find a quadratic equation in x having $(3 + 2i)$ and $(3 - 2i)$ as roots, where $i = \sqrt{-1}$.

6. Find the minimum value of $x^2 - 10x$ and illustrate your answer by means of a rough graph.

A rectangle is to be formed by bending a piece of wire 20 inches long. Find the maximum area that the rectangle can have.

7. A market gardener had 1,200 cabbage plants for sale and he tied them into bundles putting the same number of plants into each bundle. If he had put 5 plants less in each bundle, he would have had 8 more bundles. How many plants did he put in each bundle ?

8. Using the same axes and the same scales draw the graphs of $x - 2y + 2 = 0$ and $x + y = 2$ for values of x from $x = -2$ to $x = +2$.

On your graph shade in the set of ordered pairs (x, y) which simultaneously satisfy the three inequalities:-

$$x - 2y + 2 \geqslant 0; \quad x + y \leqslant 2; \quad y > \tfrac{1}{2}.$$

9. Differentiate from first principles $x^2 - x$ with respect to x.

A particle moves in a straight line so that its distance s(ft.) from a fixed point at time t(sec) is given by $s = t^2 + t$. Find the speed (rate of change of distance with respect to time) of the particle in ft. per sec. when $t = 2$.

OR

9.(i) Evaluate ${}^7C_2$, ${}^7C_5$, ${}^7P_2$, ${}^7P_5$.

(${}^nC_r$ and ${}^nP_r$ denote, respectively, the number of combinations and the number of permutations of n things taking r at a time.)

How many different sets of three books could be made from four different books ?

(ii) Write down the first four terms in the expansion of $(1 + x)^6$ and hence, or otherwise, find the value of $(1\cdot002)^6$ correct to five places of decimals.

10.(i) Write each of the following in the form a^x:-

$$\sqrt{a}, \quad \sqrt[3]{a^2}, \quad a\sqrt{a}.$$

(ii) Prove that $\log_b a = \log_c a \div \log_c b$.

Hence, or otherwise, show that

$\log_q p . \log_p q = 1$, and find the value of $\log_2 10$.

OR

10. State whether the statement "Every relation is a function" is true or false. Illustrate your answer by an example.

Find by graphical methods, or otherwise, which of the following relations are functions:
[The set of first elements of the ordered pairs is the domain. In (iii), (iv) take $A = \{5, 6, 7, 8\}$].

(i) $\{(1, 1), (2, 1), (3, 1), (4, 1)\}$,

(ii) $\{(1, 1), (1, 2), (1, 3), (1, 4)\}$,

(iii) $\{(x, y) \mid x \in A, \text{ and } y = x + 1\}$,

(iv) $\{(x, y) \mid x \in A, y \in A \text{ and } y > x\}$.