AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1966

MATHEMATICS (PASS) - PAPER I (300 marks)

WEDNESDAY, 8th JUNE - Morning, 10 to 12.30

Six questions to be answered. All questions carry equal marks.
Mathematical Tables may be obtained from the Superintendent.

1. (i) Given a line segment of length 1 unit, construct a line segment of length $\sqrt{2}$ units. Then draw a line to represent the number line, showing the points that correspond to the numbers 0, 1, $\sqrt{2}$, 2, $2\sqrt{2}$.

   (ii) Show how to divide a given line segment into two parts such that the square on one part is twice the square on the other part.

2. A right circular cone has a vertical height of 7 inches and the radius of its base is 3 inches. Find the volume of the cone, taking $3\frac{1}{7}$ for π.
If a sphere has the same volume as the cone, find the diameter of the sphere, correct to the nearest inch.

3. (i) The first term of a geometric progression is 3 and the second term is 6. Write down the third term.
Find T_n, the nth term of that progression, and S_n, the sum of the first n terms, and show that $S_n = 2T_n - 3$ for that progression.

   (ii) In a certain country the number of new cars bought in 1963 was 10,000. The number bought in 1964 was 11,000 and the number bought in 1965 was 12,100. If the number continues to increase by 10 per cent from one year to the next, find, correct to the nearest thousand, the number of new cars that will be bought in the year 1973.
[Take $2{\cdot}594$ for $(1{\cdot}1)^{10}$].

4. Prove that if two triangles are equiangular their corresponding sides are proportional.
Two chords of a circle, AB and CD, intersect at E. Prove AE.EB = CE.ED.

5. Prove that the areas of similar triangles are proportional to the squares on corresponding sides.
A triangular field ABC of area 9 acres is divided into two parts by a straight fence PQ parallel to BC. P is on AB and Q is on AC. If AP = $\frac{2}{3}$ AB, find the area of PBCQ.

6. (i) Explain, with the aid of a diagram, what is meant by an angle of one radian.
A, B are two points on the circumference of a circle centre O, such that the distance from A to B along the minor arc is equal to AO + OB. What is the size of the angle AOB ?

   (ii) If A, B are acute angles and $\sin A = \frac{4}{5}$, $\sin B = \frac{12}{13}$, find the values of $\sin(A + B)$ and $\cos(A + B)$ without using the Tables or slide-rule.
Is the angle (A + B) acute ? Explain your answer.

OR

6. What is the range of values of $\sin x$, the domain of x being the real numbers ?
Why is $\sin x$ said to be a *periodic* function ?
Draw a graph of $\sin x$ (where x is the radian measure of the angle) for values of x from 0 to 4π.
In what intervals (i.e. for what values of x) between 0 and 4π does $\sin x$ decrease as x increases ?

7. (i) Prove $\dfrac{1}{1 - \sin\alpha} + \dfrac{1}{1 + \sin\alpha} = 2\sec^2\alpha$

   (ii) Write down the cosine rule for a triangle. A,B,C are three points. If AB = 200 yards, BC = 120 yards and $\angle$ABC = 105°58', find the length of AC.

8. The line $3x + 2y = 6$ cuts the x-axis at A and the y-axis at B. Find (i) the co-ordinates of A and B, (ii) the slope of AB, (iii) the equation of the line through the origin parallel to AB, (iv) the equation of the line through the origin perpendicular to AB.

9. The co-ordinates of two points P,Q are (-1, 2), (6, 1), respectively. Find the co-ordinates of the mid-point of PQ.
Find the co-ordinates of the point R such that POQR is a parallelogram, where O is the origin.

10. (i) Show that the five numbers 2, 6, 8, 9, 10, have the same arithmetic mean as the five numbers 1, 2, 7, 12, 13.
Find either the mean deviation or the standard deviation of 2, 6, 8, 9, 10 and also of 1, 2, 7, 12, 13.

    (ii) Eighty students sat an examination. The marks awarded ranged from 0 to 5. The following frequency distribution shows the number of students who obtained the various marks:

Marks Obtained	0	1	2	3	4	5
Number of Students	7	15	26	16	11	5

Draw a frequency polygon or a histogram to represent the distribution.