AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1965

MATHEMATICS—GEOMETRY—PASS

MONDAY, 21st JUNE—Morning, 10 to 12.30

Six questions to be answered.

Mathematical Tables may be obtained from the Superintendent.

1. Prove that the perpendiculars drawn from the vertices of a triangle to the opposite sides are concurrent.

(30 marks)

2. Describe how to construct a direct common tangent to two given circles.
Given two circles and the length of a chord in one of them, show how to find a position of the chord so that when produced it is a tangent to the other circle.

(30 marks)

3. ABCDE is a regular five-sided figure inscribed in a circle. If AC cuts BD at P, show that

(i) APDE is a parallelogram,

(ii) the triangles ADC and PCD are similar,

(iii) $AC.CP = PA^2$.

(30 marks)

4. Prove that the areas of similar triangles are proportional to the squares on corresponding sides.
In a triangle ABC the angle CAB is a right-angle and AD is the perpendicular from A to BC (D is on BC). Prove that $CA^2 : AB^2 = CD : DB$.
Hence, or otherwise, show how to construct a square having an area $1\frac{1}{2}$ times the area of a given square.

(35 marks)

5. If from the vertex of a triangle a perpendicular is drawn to the base, prove that the rectangle contained by the sides of the triangle is equal to the rectangle contained by the perpendicular and the diameter of the circumcircle.
ABC is a triangle in which AB = 3AC and D is a point on the side BC. Prove that the area of the circumcircle of the triangle ABD is nine times the area of the circumcircle of the triangle ADC.

(35 marks)

6. At 4 p.m. an aeroplane travelling due West at a speed of 480 m.p.h. flies over Shannon airport. Ten minutes later another aeroplane travelling at a speed of 800 m.p.h. in a direction 35° North of West flies over the airport. If both aeroplanes keep to the same horizontal plane, how far apart are they at 5.25 p.m. and what is then the direction of the faster aeroplane from the other ?
Find also the distance between the aeroplanes when they are equally distant from the airport.

(35 marks)

7. Prove that $\sin(A + B) = \sin A\cos B + \cos A\sin B$, where A, B and (A + B) are acute angles.

Deduce that $\sin(A + B) = \frac{77}{85}$ where $4\sin A = 3\sin(90° - A)$ and $\tan B = \frac{8}{15}$.

(35 marks)