AN ROINN OIDEACHAIS.

(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1957.

MATHEMATICS—GEOMETRY—PASS.

THURSDAY, 6th JUNE.—MORNING, 10 TO 12.30.

Six questions to be answered.

Mathematical Tables may be obtained from the Superintendent.

1. Prove that the sum of the squares on two sides of a triangle is equal to twice the square on half the third side together with twice the square on the median that bisects the third side.

In a triangle ABC the base BC is fixed in magnitude and position and AB^2+AC^2 is constant: find the locus of A.

[30 marks.]

2. Prove that if a straight line is drawn parallel to one side of a triangle it cuts the other sides proportionally.

P is any point between the arms of an angle ABC. Show, with proof, how to draw through P a line QPR so that QP=2PR, where Q lies on AB and R lies on BC.

[30 marks.]

3. If two triangles have the same altitude, prove that their areas are proportional to the lengths of their bases.

O is a point within a triangle ABC so that AO produced bisects BC and BO produced meets AC at E, so that AE=2EC. CO produced meets AB at F.

Show that $\triangle$ABO and $\triangle$AOC are equal in area. Find also the ratio of AF to FB.

[30 marks.]

4. A straight line PT touches a circle at T and a secant through P meets the circle at A and B. Prove that $PT^2=PA . PB$.

Show, with proof, how to draw a circle passing through two given points and touching a given straight line.

[35 marks.]

5. Prove that the areas of similar polygons are proportional to the squares on their corresponding sides.

Show how to construct an equilateral triangle equal in area to the sum of two given equilateral triangles. Give proof.

[35 marks.]

6. In a triangle ABC, using the usual notation, prove that

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}.$$

From a ship sailing due East at 10 knots, a lighthouse bears 20°36′ North of East at noon, and 39°10′ North of West at 1.30 p.m. Find in yards, as accurately as the tables allow,

(i) the distance of the lighthouse from the ship at 1.30 p.m. ;

(ii) the distance between the ship and the lighthouse when they are nearest one another.

(1 knot=1·152 miles per hour.)

[35 marks.]

7. Prove that $\cos(A+B)=\cos A\cos B-\sin A\sin B$, where A, B, A+B, are acute angles.

Deduce that $\cos 2A=\cos^2 A-\sin^2 A=1-2\sin^2 A$.

In a triangle ABC, using the usual notation, prove that

$$\frac{\cos 2A}{a^2}-\frac{\cos 2B}{b^2}=\frac{1}{a^2}-\frac{1}{b^2}.$$

[35 marks.]