AN ROINN OIDEACHAIS
(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1957.

MATHEMATICS—Algebra—Pass.

TUESDAY, 11*th* *JUNE*.—Morning, 10 to 12.30.

All questions to be answered.

Mathematical Tables may be obtained from the Superintendent.

---

1. If $x+\frac{1}{x}=y$, show that $x^3+\frac{1}{x^3}=y^3-3y$, and express $x^5+\frac{1}{x^5}$ in terms of y.

[25 marks.]

2. (*a*) Show that $x+3$ is a factor of $x^3+5x^2-2x-24$, and find the other factors. Hence write down two algebraic expressions of the second degree of which $x+3$ is the H.C.F. and $x^3+5x^2-2x-24$ is the L.C.M.

(*b*) Show that

$a+b+c$ is a factor of $ab(a^2-b^2)+bc(b^2-c^2)+ca(c^2-a^2)$.

[25 marks.]

3. Solve the equations

(*a*) $6x-11\sqrt{x}-35=0$.

(*b*) $$\begin{cases} x+\frac{1}{y}=2, \\ y+\frac{1}{x}=8. \end{cases}$$

[30 marks.]

4. Show that the sum to n terms of the A.P. $a, a+d, a+2d, \ldots\ldots$ is $\frac{n}{2}\left\{2a+(n-1)d\right\}$.

An A.P. consisting of 15 terms has 23 as its middle term : find the sum of the series.

Show also that seven times the sum of the odd terms is eight times the sum of the even terms.

[30 marks.]

5. Show that $a^m \times a^n = a^{m+n}$, and that $(a^m)^n = a^{mn}$.

(i) Solve the equation $2^{2x} - 10(2^x) + 16 = 0$.

(ii) Write in simplest form (*a*) $\log_2 \frac{1}{8}$, (*b*) $\log_2 \sqrt[3]{4}$, (*c*) $a^{\log_a x}$.

[30 marks.]

6. XYZ is an acute angle. From a point A on XY a perpendicular AB is drawn to YZ. From B a perpendicular BC is drawn to XY. From C a perpendicular CD is drawn to YZ. From D a perpendicular DE is drawn to XY, and so on. Show that AB, BC, CD, DE . . . . form a geometrical series.

If $\angle XYZ = 30^\circ$ and if AY$=20$, find the *n*th term of the series. Find, also, the least value of *n* for which the *n*th term is less than 0·5.

[30 marks.]

7. Draw the graph of the function $(2+x)^2(4-x)$ for values of x from -3 to $+4$.

Using your graph, solve the equations :—

(i) $(2+x)^2(4-x) = 4$;

(ii) $(2+x)^2(4-x) = 2x + 5$.

[30 marks.]