AN ROINN OIDEACHAIS.

(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1955.

MATHEMATICS—GEOMETRY—PASS.

FRIDAY, 10*th JUNE*.—Morning, 10 to 12.30.

Six questions to be answered.

Mathematical Tables may be obtained from the Superintendent.

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1. Prove that the rectangle contained by the diagonals of a quadrilateral inscribed in a circle is equal to the sum of the two rectangles contained by its opposite sides.

[30 marks.]

2. In a right-angled triangle prove that if a perpendicular is drawn from the right angle to the hypotenuse, the triangles on each side of it are similar to the whole triangle and to one another.

In a triangle ABC, the angle B is a right angle and BD is perpendicular to AC. Prove (i) that BD is a mean proportional between AD and DC, (ii) that AD is a third proportional to CA and AB.

[30 marks.]

3. Show, with proof, how to construct an isosceles triangle having each of the base angles double the vertical angle.

[30 marks.]

4. A triangle ABC is inscribed in a circle. Tangents to the circle at B and C meet at S. With centre S and radius SB a circle is drawn which meets AC produced at D. Prove that the angle CDS is equal to the angle ABC. If DS produced meets AB at P, prove that P lies on the circle BCD.

[35 marks.]

5. Show, with proof, how to divide the base BC of a triangle ABC internally and externally in the ratio of the sides BA, AC.

If D, E, are the points of section, prove that the angle DAE is a right angle, and hence find the locus of the vertex of a triangle ABC in which the base BC is fixed in magnitude and position and the other two sides are in a given ratio.

[35 marks.]

6. In a triangle ABC, using the usual notation, prove that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

In a triangle PQR the angle PQR is 40°, the angle PRQ is 35°, and QR is 3 inches in length. Calculate (i) the length of PQ, (ii) the area of the triangle, (iii) the radius of the circumscribed circle.

[35 marks.]

7. Prove that $\sin(A+B)=\sin A\cos B+\cos A\sin B$, when A, B, (A+B) are acute angles.

Show that $\sin 2A=2\sin A\cos A=\frac{2\tan A}{1+\tan^2 A}$,

and that $\tan 15°=2-\sqrt{3}$.

[35 marks.]