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(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1954.

MATHEMATICS—GEOMETRY—PASS.

FRIDAY, 11*th* *JUNE*.—Morning, 10 to 12.30.

Six questions to be answered.

Mathematical Tables may be obtained from the Superintendent.

1. Prove that the opposite angles of a cyclic quadrilateral are together equal to two right angles.

Two circles intersect at A and B. P and Q, two points on the circumference of one of the circles, are joined to A. PA and QA, produced, meet the circumference of the second circle at H and K, respectively. HK and QP, produced, meet at X. Prove that XPBH is a cyclic quadrilateral.

[30 marks.]

2. The bisector of the angle BAC of a triangle ABC meets BC at D. Prove that AB : AC = BD : DC.

State and prove the converse theorem.

[30 marks.]

3. The base of a triangle is fixed in magnitude and position and the vertical angle is constant in magnitude. Find the locus of (*a*) the vertex, (*b*) the orthocentre, and (*c*) the centre of the incircle of the triangle.

[30 marks.]

4. If the vertical angle of a triangle is bisected by a straight line which cuts the base, prove that the rectangle contained by the sides of the triangle is equal to the rectangle contained by the segments of the base together with the square on the straight line which bisects the vertical angle.

In a triangle ABC, the bisector AD of the angle BAC meets the base BC in D. If AB=7 cms., BC=6 cms., CA=5 cms., find the length of AD.

[35 marks.]

5. Show, with proof, how to construct a pentagon similar to a given pentagon and such that its area will be four-fifths the area of the given pentagon.

[35 marks.]

6. In a triangle ABC, using the usual notation, prove that

$$a^2=b^2+c^2-2bc \cos A.$$

If the sides of a triangle are in the ratio 4 : 5 : 7, find the greatest angle. [35 marks.]

7. Using the usual notation, prove that the area of a triangle is $\frac{1}{2}ab \sin C$.

A regular polygon of n sides is inscribed in a given circle of radius r. Show that the area of the polygon is $\frac{1}{2}nr^2 \sin \frac{360°}{n}$.

Find the ratio of the areas of two regular hexagons, one of which is inscribed in a given circle and the other circumscribed about it. [35 marks.]