AN ROINN OIDEACHAIS.

(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1953.

MATHEMATICS—GEOMETRY—PASS.

THURSDAY, 11th JUNE.—Morning, 10 to 12.30.

Six questions to be answered.

Mathematical Tables may be obtained from the Superintendent.

1. Two chords of a circle, AB and CD, intersect internally at L. Prove that AL.LB=CL.LD.

PQ is a diameter that does not pass through L. Show, with proof, how to find a point R in PQ such that PR.RQ=AL.LB.

[33 marks.]

2. In a triangle ABC the angle C is obtuse, and D is the foot of the perpendicular from A to BC produced. Prove that

$$AB^2=BC^2+CA^2+2BC.CD.$$

If M is the mid-point of BC, prove that

$$AB^2+CA^2=2AM^2+2BM^2.$$

[33 marks.]

3. Show, with proof, how to inscribe a circle in a given triangle.

D is the centre of the circle inscribed in a triangle ABC, and AD is produced to meet the circumscribed circle at O. Prove that

$$OB=OC=OD.$$

[33 marks.]

4. When are rectilinear figures said to be "*similar*"?

Prove that the areas of two similar quadrilaterals are to each other as the squares on any pair of corresponding sides.

[The corresponding Theorem for triangles may be assumed.]

[33 marks.]

5. Prove that the rectangle contained by the diagonals of a cyclic quadrilateral is equal to the sum of the two rectangles contained by its opposite sides.

An equilateral triangle ABC is inscribed in a circle, and P is any point on the arc AB. Prove that PA+PB=PC.

[33 marks.]

6. In a triangle ABC, prove that

(i) $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$;

(ii) $c = a \cos B + b \cos A$,

where a, b, c are the lengths of the sides BC, CA, AB respectively.

Hence, or otherwise, prove that

$$\sin(A+B) = \sin A \cos B + \cos A \sin B.$$

[34 marks.]

7. In a quadrilateral ABCD the lengths of the sides AB, BC, CD are 5″, 3″ and 2″ respectively. The angle ABC=90°, and the angle BCD=110°. Calculate the length of the side AD.

[34 marks.]