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(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1949.

MATHEMATICS.—GEOMETRY.—PASS.

THURSDAY, 9th JUNE.—MORNING, 10 TO 12.30.

Six **questions to be attempted.**

All questions are of equal value.

Mathematical Tables may be obtained from the Superintendent.

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1. Show, with proof, how to inscribe an equilateral triangle in a given circle.

If r be the radius of the circle and s the side of the equilateral triangle, show that $s=r\sqrt{3}$.

2. Show how to draw (i) a direct common tangent, (ii) a transverse common tangent, to two given circles.

Prove your construction in *one* case.]

If X, Y are the centres of the circles, prove that the transverse common tangent divides XY in the ratio of the radii of the circles.

3. M is the mid-point of the side BC of the triangle ABC: prove that

$$AB^2+AC^2=2(AM^2+BM^2).$$

(i) P and Q are fixed points, and a point X moves so that XP^2+XQ^2 is constant: find the locus of X.

4. Show, with proof, how to divide a straight line into two parts so that the square on one part shall be equal to twice the square on the other part.

5. Show how to divide a straight line AB internally at P so that $AP^2=AB.PB$. Give proof.

If a triangle whose sides are equal respectively to AB, AB, AP be constructed, prove that its angles will be in the ratios 2 : 2 : 1.

Or,

5. Prove that the areas of similar triangles are proportional to the squares on their corresponding sides.

(i) Show how to bisect the area of a triangle by a straight line drawn parallel to one of its sides.

6. Prove that

$$\sin(A+B)=\sin A\cos B+\cos A\sin B,$$

where A, B are acute angles and $(A+B)<90°$.

Express $\sin 75°$ in its simplest surd form.

Or,

6. (i) In a triangle ABE, $AB=a$ and the angles A, B are α, β respectively. Show that

$$AE=\frac{a\sin\beta}{\sin(\alpha+\beta)}.$$

(ii) If in (i) above, E represents an aeroplane and A a point of observation from which the angle of elevation of E is θ, show that the height of the aeroplane above the horizontal plane through A is

$$a\sin\beta\sin\theta\operatorname{cosec}(\alpha+\beta).$$

7. The bisector of the angle A of a triangle ABC meets BC at D. BC=10 ins., CA=7 ins., AB=5 ins. Find the number of degrees in each angle of ABC and the length of AD.

8. Draw the graph of the expression

$$\sin\theta+2\cos\theta \text{ from } \theta=0° \text{ to } \theta=360°.$$

From your graph find approximately the maximum and the minimum values of the expression and the corresponding values of θ.