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(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1948.

MATHEMATICS.—GEOMETRY.—PASS.

WEDNESDAY, 16*th* *JUNE*.—Morning, 10 to 12.30.

Six questions to be attempted, of which not more than *four* may be selected from Section A.

All questions are of equal value.

Mathematical Tables may be obtained from the Superintendent.

Section A.

1. Prove that the angle which a tangent to a circle makes with a chord through the point of contact is equal to the angle in the alternate segment.

PR and PS are tangents to a circle from a point P, and PN, the perpendicular from P on RS, cuts the minor arc RS at C. If C is the circumcentre of the triangle PRS, show that PRS is equilateral.

2. Prove that the vertices of a regular octagon are concyclic.

If r is the radius of the circumcircle, prove that the area of the octagon is $2\sqrt{2}r^2$. (A trignometrical solution will be accepted.)

3. Prove that the internal bisector of the vertical angle of a triangle divides the base in the ratio of the other two sides.

ABC is a triangle and the internal bisector of the angle A meets BC at D. If AB=12, BC=8, CA=10, calculate the lengths of BD and DC.

4. Prove that equiangular triangles are similar.

PAB and PCD are two secants drawn from an external point P to cut a circle at A, B and C, D. Show that the triangles PBC, PDA are similar and hence prove that PA.PB=PC.PD.

5. ABC is a triangle in which A=90°. The altitude through A meets BC in D. Prove that CD is a third proportional to BC and CA.

Hence, or otherwise, show how to construct the third proportional to two given straight lines.

6. A fixed point P, outside a given circle is joined to any point Q on the circumference. Find the locus of the mid point of PQ.

Section B.

7. From A and B, two points a feet apart which lie in a horizontal plane on the same side of and in the same straight line as the foot of a vertical tower (B being nearer the tower than A), the angles of elevation of the top of the tower are α and β respectively.

Show that the height of the tower, h, is given by

$$h = a \sin\alpha \sin\beta \operatorname{cosec}(\beta - \alpha).$$

Using your tables calculate h when $a = 157$ ft., $\alpha = 20^\circ\ 18'$, $\beta = 31^\circ\ 29'$.

8. ABC is a triangle in which $b = 8$, $c = 6$, $A = 67^\circ\ 15'$. Calculate the side a and the length of the median through A.

9. (i) Evaluate

$$\cos A + \cos(120^\circ + A) + \cos(240^\circ + A).$$

(ii) Prove that

$$\sin 2A = \frac{2 \tan A}{1 + \tan^2 A}.$$

Or

Draw the graphs of $y = \sin 2x$, $y = 1 - \cos x$, for values of x from 0° to 180° inclusive.

From your graphs state the changes in each of the functions $\sin 2x$ and $(1 - \cos x)$ as x varies from 0° to 180°.

Also find a value of x between 0° and 180° which satisfies the equation

$$\sin 2x + \cos x = 1.$$