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(Department of Education).

LEAVING CERTIFICATE EXAMINATION, 1943.

MATHEMATICS—Algebra—Pass.

MONDAY, 21st JUNE.—MORNING 10 TO 12.30.

Seven questions may be answered.

Mathematical Tables may be obtained from the Superintendent.

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1. Find the values of yz, zx and xy which satisfy the equations

$$3yz-2zx+xy=1,$$
$$2yz+zx+2xy=4,$$
$$yz+zx-2xy=-17.$$

Hence find the values of x, y, z which satisfy the equations.

[28 marks.]

2. Solve the following simultaneous equations for x and y and verify your answers:

$$2x^2+4xy+3y^2=17,$$
$$2x+3y=1.$$

[28 marks.]

3. If $t=x+1/x$ prove that $t^2-2=x^2+1/x^2$.
Dividing across the equation

$$6x^4-5x^3-38x^2-5x+6=0$$

by x^2, express the left-hand side in terms of t and hence solve the equation completely.

[28 marks.]

4. If $x=3+2\sqrt{3}$ is a root of the equation

$$x^2+2(a+b\sqrt{3})x=26(a+b\sqrt{3})-7,$$

where a and b are rational numbers, determine a and b. Verify that the second root of the equation is $x=-7-4\sqrt{3}$.

[28 marks.]

5. Factorize completely

(i) $2x^3+5x^2-9x-18$;

(ii) $(2x-a-b)^3-(x-a)^3-(x-b)^3$.

[28 marks.]

6. (i) Find the values of a and b for which

$$\log_{10}(2a+b)=2,$$
$$\log_{10}(a+3b)=3.$$

(ii) Find x and y if

$$(\sqrt{2})^x=y\sqrt{2}=\tfrac{1}{2}.$$

[29 marks.]

7. A rectangular piece of tin is 750 sq. in. in area and is such that a square of side 8 in. may be cut from each corner of it and the edges of the remainder turned up so as to form an open rectangular box whose volume is 1,008 cu. in. Find the dimensions of the piece of tin.

[29 marks.]

8. (i) Find the values of x for which x, $2x^2$ and $3x^3$ are the first three terms in an arithmetical progression. In each case evaluate the sum of the first ten terms in the progression.

(ii) In the case of each of the following series write down the sixth term :

(*a*) $1+3x+5x^2+7x^3+9x^4$ . . .

(*b*) $1-4+9-16+25$ . . .

(*c*) $2+5+10+17+26$ . . .

(*d*) $1+3+6+10+15$ . . .

[29 marks.]

9. The first three terms in a geometric progression are $x+9$, $x+3$ and x. Find x and the sum of the first six terms of the series.

Find also the least number of terms of the series that must be taken so that their sum exceeds 23·99.

Or

The volume of a right circular cylinder is 200 cu. in. and its radius is r in. Find an expression for S, the total surface area, in terms of r. Using 3·14 as the value of π, complete the following table :

r	2	2·5	3	3·5	4	4·5	5
S	225		190	191		216	

Construct a graph showing the relation between S and r and estimate from the graph the least value of S and the corresponding value of r. Find also the values of r for which the surface area does not exceed 210 sq. in.

[29 marks.]