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(Department of Education.)

LEAVING CERTIFICATE EXAMINATION, 1947.

MATHEMATICS.—GEOMETRY.—PASS.

WEDNESDAY, 11*th JUNE*.—Morning, 10 to 12.30.

Six questions to be attempted, of which not more than *four* may be selected from Section A.

All questions are of equal value.

Mathematical Tables may be obtained from the Superintendent.

Section A.

1. In a triangle ABC, D is the mid-point of BC; prove that

$$AB^2+AC^2=2AD^2+2BD^2.$$

If the base of a triangle is given in magnitude and position and the sum of the squares on the other two sides is known, find the locus of the vertex.

2. Show, with proof, how to describe a regular pentagon about a given circle.

If the radius of the circle is one inch, calculate the length of the side of the pentagon, giving your answer correct to three significant figures. (A trigonometrical solution will be accepted.)

3. Prove that a line drawn parallel to one side of a triangle divides the other two sides proportionally. (One case will suffice.)

Three parallel lines cut two other straight lines. Prove that the intercepts on these two lines are in proportion.

4. Prove that the areas of similar triangles are proportional to the squares on a pair of corresponding sides.

ABC is a triangle and PQ is drawn parallel to BC, meeting AB, AC at P, Q respectively. If the area of APQ is equal to half ABC, find the value of the ratio AP : PB.

5. P is a point outside a circle; PT is a tangent to the circle and the straight line PAB cuts the circle at A, B. Using similar triangles, or otherwise, prove that $PT^2=PA.PB$.

Show, without proof, how to describe a circle passing through two given points and touching a given straight line.

6. Similar and similarly situated quadrilaterals are described on the sides of a right-angled triangle; prove that the quadrilateral on the hypotenuse is equal to the sum of the other two quadrilaterals.

Section B.

7. In a triangle ABC, $a=54{\cdot}23$, $b=62{\cdot}14$, $c=57{\cdot}57$; calculate A and the area of the triangle.

8. Show that $\tan^2\theta=\sec^2\theta-1$.

Find the angle between 0° and 90° which satisfies the equation

$$2\tan^2\theta+3\sec\theta=7.$$

9. Using the expansions of sin (A+B), cos (A+B), show that

(i) $\sin 75^\circ=\frac{1}{4}(\sqrt{6}+\sqrt{2})$,

(ii) $\cos 75^\circ=\frac{1}{4}(\sqrt{6}-\sqrt{2})$,

(iii) $\tan 75^\circ=2+\sqrt{3}$.

10. Draw the graphs of

(i) $y=\cos 2x$, (ii) $y=2\sin x$,

for values of x lying between 0° and 180°.

Use your graphs to find two angles between 0° and 180° which satisfy the equation

$$\cos 2x=2\sin x.$$